Contents

Preface v

Contributors xv

1 Jackson Network Models of Manufacturing Systems
John A. Buzacott, J. George Shanthikumar, and David D. Yao 1
1.1 Introduction . . . 1
1.2 Jackson Networks . . . 4
1.2.1 The Open Model . . . 5
1.2.2 The Closed Model . . . 8
1.2.3 The Semi-Open Model . . . 9
1.3 The Throughput Function and Computation . . . 11
1.4 Monotonicity of the Throughput Function . . . 15
1.4.1 Equilibrium Rate . . . 15
1.4.2 PF_2 Property . . . 16
1.4.3 Likelihood Ratio Ordering . . . 17
1.4.4 Shifted Likelihood Ratio Ordering . . . 20
1.5 Concavity and Convexity . . . 24
1.6 Multiple Servers . . . 27
1.7 Resource Sharing . . . 30
1.7.1 Aggregation of Servers . . . 30
1.7.2 Aggregation of Nodes . . . 32
1.8 Arrangement and Majorization . . . 35
1.9 Conclusions . . . 39
1.10 Notes . . . 40
1.11 References . . . 43

2 Hierarchical Modeling of Stochastic Networks, Part I: Fluid Models
Hong Chen and Avi Mandelbaum 47
2.1 Introduction . . . 47
2.1.1 Macro, Meso and Microscopic Models for an i.i.d. Sequence . . . 50
2.1.2 Strong Approximations - A Unifying Framework . . . 53
2.1.3 Summary . . . 55
2.2 A Flow Network in Discrete Time . . . 55
2.2.1 The Microscopic Model and Its Dynamics . . . 56

2.2.2 Reformulation in Terms of Cumulants and Oblique Reflection . . . 58
2.2.3 Mesoscopic Models and Strong Approximations . . . 61
2.2.4 Macroscopic Models: FSLLN's . . . 63
2.2.5 Deviations Between Micro and Macro Models: FCLT 64
2.3 Flow Networks in Continuous Time . . . 65
2.3.1 Flow Networks with Time Inhomogeneous Dynamics 65
2.3.2 State-Dependent Dynamics . . . 66
2.4 Linear Fluid Network and Bottleneck Analysis . . . 68
2.4.1 Traffic Equations and Bottleneck Definitions . . . 68
2.4.2 Bottleneck Analysis . . . 71
2.5 Functional Strong Law of Large Numbers . . . 74
2.5.1 FSLLN's for Nonlinear Fluid Networks . . . 74
2.5.2 FSLLN's for Nonparametric Jackson Queueing Networks . . . 75
2.5.3 FSLLN's for State-Dependent Networks . . . 78
2.6 Applications and Hints at Prospects of Fluid Models . . . 82
2.6.1 Stochastic Fluid Models for Manufacturing and Communication Systems . . . 82
2.6.2 Heterogeneous Fluid Networks: Bottleneck Analysis and Scheduling Control . . . 85
2.6.3 Transient Analysis of the $M_t/M_t/1$ Queue . . . 92
2.7 References and Comments . . . 97
2.8 References . . . 100

3 Hierarchical Modeling of Stochastic Networks, Part II: Strong Approximations
Hong Chen and Avi Mandelbaum **107**
3.1 Introduction . . . 107
3.2 The Model . . . 108
3.2.1 Primitives and Dynamics . . . 108
3.2.2 Underlying Assumptions and Parameters . . . 108
3.2.3 Nonparametric Jackson Networks . . . 109
3.3 Preliminaries . . . 111
3.3.1 Traffic Equations and Bottlenecks . . . 111
3.3.2 The Oblique Reflection Mapping . . . 111
3.3.3 Reflected Brownian Motion on the Orthant . . . 112
3.4 The Main Results . . . 112
3.4.1 Functional Strong Approximations . . . 113
3.4.2 Functional Laws of the Iterated Logarithm . . . 114
3.4.3 FSLLN's and Fluid Approximations . . . 114
3.4.4 FCLT's and Diffusion Approximations . . . 115
3.5 Fitting Parametes . . . 116
3.5.1 Nonparametric Jackson Networks . . . 116
3.5.2 Product Form and Single Station . . . 117

3.6 Proof of the Main Results 117
3.7 References, Possible Extensions and Future Research 125
3.8 References 128

4 A GSMP Framework for the Analysis of Production Lines
Paul Glasserman and David D. Yao **133**
4.1 Introduction 133
4.2 GSMP and Its Scheme 135
4.2.1 The Scheme: GSMS 135
4.2.2 Language and Score Space 136
4.3 Structural Properties of the Scheme 138
4.3.1 Some Useful Properties 138
4.3.2 Condition **(M)** 140
4.3.3 Condition **(CX)** 143
4.3.4 Minimal Elements 145
4.3.5 Monotonicity and Convexity 147
4.3.6 Characteristic Function 148
4.3.7 Subschemes 150
4.3.8 Synchronized Schemes 152
4.4 The (a, b, k) Tandem Queue 154
4.4.1 Production Lines Under Kanban Control 154
4.4.2 Properties with Respect to Service Times 156
4.5 Properties with Respect to (a, b, k) 160
4.5.1 Monotonicity with Respect to (a, b, k) 160
4.5.2 Concavity with Respect to (a, b, k) 162
4.6 Line Reversal 164
4.6.1 Reversibility of Departure Epochs 164
4.6.2 Full Reversibility 167
4.7 Subadditivity and Ergodicity 168
4.7.1 Event-Epoch Vectorization 169
4.7.2 The Subadditive Ergodic Theorem 170
4.7.3 More General Matrices 172
4.8 Cycle Time Limits 176
4.8.1 Existence of the Limits 176
4.8.2 Rate of Convergence 179
4.9 Notes 182
4.10 References 185

5 Stochastic Convexity and Stochastic Majorization
Cheng-Shang Chang, J. George Shanthikumar, and David D. Yao **189**
5.1 Introduction 189
5.2 Stochastic Order Relations: Functional Characterizations . 192
5.3 Second-Order Stochastic Properties 196

5.3.1 Stochastic Convexity 196
5.3.2 Stochastic Supermodularity and Submodularity . . . 200
5.3.3 Markov Chain Applications 203
5.3.4 A Joint Setup Problem 205
5.3.5 Production with Trial Runs 207
5.4 Arrangement and Likelihood Ratio Orderings 210
5.4.1 The Connection 211
5.4.2 Queueing Network Applications 213
5.5 Stochastic Rearrangement and Majorization 215
5.5.1 The Deterministic Theory 215
5.5.2 The Stochastic Counterpart 216
5.5.3 Connections to Stochastic Convexity and Stochastic Supermodularity 223
5.6 Notes 227
5.7 References 229

6 Perturbation Analysis of Production Networks
Paul Glasserman **233**
6.1 Introduction 233
6.2 Overview Through the Single-Machine Model 235
6.3 Differentiation 243
6.3.1 Classes of Random Functions 243
6.3.2 Differentiability of Inputs 247
6.3.3 Differentiability of Recursions 249
6.4 Analysis of the Single-Machine Model 252
6.5 Production Networks 256
6.5.1 The Production Line 256
6.5.2 Finite Buffers 258
6.5.3 Implementation 261
6.5.4 A Kanban System 262
6.5.5 Systems with Rework and Scrap 263
6.5.6 A System with Alternative Sourcing 265
6.5.7 A System with Subassemblies 266
6.6 Steady-State Derivative Estimation 268
6.6.1 Discrete Time 268
6.6.2 Continuous Time 272
6.7 Concluding Remarks 275
6.8 Notes 275
6.9 References 278

7 Scheduling Networks of Queues: Heavy Traffic Analysis of a Bi-Criteria Problem
Lawrence M. Wein **281**
7.1 Introduction 281
7.2 A Single Server Queue 286

7.2.1 The Scheduling Problem 286
7.2.2 The Limiting Control Problem 287
7.2.3 The Workload Formulation 289
7.2.4 Solution to the Workload Formulation 290
7.2.5 Interpreting the Solution to the Workload Formulation 292
7.3 A Closed Network 293
7.3.1 The Workload Formulation 294
7.3.2 The $c \to \infty$ Case 295
7.3.3 The $c = 0$ Case 297
7.3.4 The Bi-Criteria Case 298
7.4 A Network with Controllable Inputs 299
7.4.1 The Workload Formulation 299
7.4.2 Solution to the Workload Formulation 300
7.5 An Example 304
7.6 A Review of Related Results 315
7.6.1 Open Networks 315
7.6.2 Closed Networks 316
7.6.3 Networks with Controllable Inputs 317
7.6.4 Networks with Discretionary Routing 317
7.6.5 Production/Inventory Systems 318
7.6.6 Weak Convergence Results 318
7.7 References 319

8 Scheduling Manufacturing Systems of Re-Entrant Lines **325**
P. R. Kumar
8.1 Introduction 325
8.2 Re-Entrant Lines: The Models 327
8.3 Fluctuation Smoothing Scheduling Policies to Reduce Variance of Lateness, Variance of Cycle–Time, and Mean Cycle–Time 329
8.4 Stability of LBFS, SRPTS, EA, EDD and All Least Slack Scheduling Policies 334
8.5 Dynamic Scheduling of a Single Machine with Set-Up Times: A Push Model 345
8.6 Clear-A-Fraction Policies 347
8.7 A Lower Bound on Optimal Cost 349
8.8 A Good CAF Policy 352
8.9 Non-Acyclic Manufacturing Systems with Set-Up Times . . 353
8.10 Concluding Remarks 356
8.11 Notes 357
8.12 References 358