

Contents

Prologue	1
1 The Markov–Switching Vector Autoregressive Model	6
1.1 General Introduction	6
1.2 Markov-Switching Vector Autoregressions	10
1.2.1 The Vector Autoregression	10
1.2.2 Particular MS–VAR Processes	13
1.2.3 The Regime Shift Function	14
1.2.4 The Hidden Markov Chain	16
1.3 The Data Generating Process	17
1.4 Features of MS-VAR Processes and Their Relation to Other Non-linear Models	20
1.4.1 Non-Normality of the Distribution of the Observed Time Series	21
1.4.2 Regime-dependent Variances and Conditional Heteroskedasticity	23
1.4.3 Regime-dependent Autoregressive Parameters: ARCH and Stochastic Unit Roots	24
1.5 Conclusion and Outlook	26
1.A Appendix: A Note on the Relation of SETAR to MS-AR Processes	27
2 The State-Space Representation	29
2.1 A Dynamic Linear State-Space Representation for MS-VAR Processes	30
2.1.1 The Gaussian Measurement Equation	32
2.1.2 The Non–Normal VAR(1)–Representation of the Hidden Markov Chain	32
2.1.3 Linearity of the State-Space Representation	33

2.1.4	Markov Property of the State-Space Representation	34
2.2	Specification of the State-Space Representation	37
2.3	An Unrestricted State-Space Representation	40
2.4	Prediction-Error Decomposition and the Innovation State-Space Form	41
2.5	The MS-VAR Model and Time-Varying Coefficient Models	44
3	VARMA-Representation of MSI-VAR and MSM-VAR Processes	47
3.1	Linearly Transformed Finite Order VAR Representations	48
3.2	VARMA Representation Theorems	53
3.2.1	VARMA Representation of Linearly Transformed Finite Order VAR Representations	53
3.2.2	ARMA Representation of a Hidden Markov Chain	54
3.2.3	VARMA Representations of MSI(M)-VAR(0) Processes ..	54
3.2.4	VARMA Representations of MSI(M)-VAR(p) Processes ..	55
3.2.5	VARMA Representations of MSM(M)-VAR(p) Processes ..	56
3.3	The Autocovariance Function of MSI-VAR and MSM-VAR Processes	57
3.3.1	The ACF of the Regime Generating Process	58
3.3.2	The ACF of a Hidden Markov Chain Process	59
3.3.3	The ACF of MSM-VAR Processes	60
3.3.4	The ACF of MSI-VAR Processes	62
3.4	Outlook	64
4	Forecasting MS-VAR Processes	65
4.1	MSPE-Optimal Predictors	66
4.2	Forecasting MSM-VAR Processes	68
4.3	Forecasting MSI-VAR Processes	71
4.4	Forecasting MSA-VAR Processes	72
4.5	Summary and Outlook	75
5	The BLHK Filter	77
5.1	Filtering	78
5.2	Smoothing	82
5.A	Supplements	87
5.A.1	Conditional Moments of Regime	87
5.A.2	A Technical Remark on Hidden Markov-Chains: The MSI/MSIH(M)-VAR(0) Model	88

6 Maximum Likelihood Estimation	89
6.1 The Likelihood Function	90
6.2 The Identification Problem	92
6.3 Normal Equations of the ML Estimator	95
6.3.1 Derivatives with Respect to the VAR Parameters	96
6.3.2 Derivatives with Respect to the Hidden Markov-Chain Parameters	97
6.3.3 Initial State	99
6.4 The EM Algorithm	101
6.4.1 Estimation of γ	103
6.4.2 Estimation of σ under Homoskedasticity	107
6.4.3 Estimation of σ under Heteroskedasticity	108
6.4.4 Convergence Criteria	109
6.5 Extensions and Alternatives	110
6.5.1 The Scoring Algorithm	111
6.5.2 An Adaptive EM Algorithm (Recursive Maximum Likelihood Estimation)	113
6.5.3 Incorporating Bayesian Priors	115
6.5.4 Extension to General State-Space Models with Markovian Regime Shifts	116
6.6 Asymptotic Properties of the Maximum Likelihood Estimator	118
6.6.1 Asymptotic Normal Distribution of the ML Estimator	118
6.6.2 Estimation of the Asymptotic Variance-Covariance Matrix	120
6.7 Conclusion	122
7 Model Selection and Model Checking	123
7.1 A Bottom-up Strategy for the Specification of MS-VAR Models	124
7.2 ARMA Representation Based Model Selection	129
7.3 Model Checking	131
7.3.1 Residual Based Model Checking	132
7.3.2 The Coefficient of Determination	133
7.4 Specification Testing	134
7.4.1 Likelihood Ratio Tests	135
7.4.2 Lagrange Multiplier Tests	135
7.4.3 Wald Tests	137
7.4.4 Newey-Tauchen-White Test for Dynamic Misspecification	139
7.5 Determination of the Number of Regimes	141

7.6	Some Critical Remarks	144
8	Multi-Move Gibbs Sampling	145
8.1	Bayesian Analysis via the Gibbs Sampler	147
8.2	Bayesian Analysis of Linear Markov-Switching Regression Models	149
8.3	Multi-Move Gibbs Sampling of Regimes	152
8.3.1	Filtering and Smoothing Step	153
8.3.2	Stationary Probability Distribution and Initial Regimes ...	154
8.4	Parameter Estimation via Gibbs Sampling	155
8.4.1	Hidden Markov Chain Step	155
8.4.2	Inverted Wishart Step	157
8.4.3	Regression Step	159
8.5	Forecasting via Gibbs Sampling	163
8.6	Conclusions	165
9	Comparative Analysis of Parameter Estimation in Particular MS-VAR Models	167
9.1	Analysis of Regimes	169
9.2	Comparison of the Gibbs Sampler with the EM Algorithm	171
9.3	Estimation of VAR Parameters for Given Regimes	172
9.3.1	The Set of Regression Equations	172
9.3.2	Maximization Step of the EM Algorithm.....	174
9.3.3	Regression Step of the Gibbs Sampler	177
9.3.4	MSI Specifications.....	179
9.3.5	MSM Specifications	181
9.4	Summary	183
9.A	Appendix: Tables	184
10	Extensions of the Basic MS-VAR Model	199
10.1	Systems with Exogenous Variables	199
10.2	Distributed Lags in the Regime	202
10.2.1	The MSI(M, q)-VAR(p) Model	202
10.2.2	VARMA Representations of MSI(M, q)-VAR(p) Processes	203
10.2.3	Filtering and Smoothing	205
10.3	The Endogenous Markov-Switching Vector Autoregressive Model	205
10.3.1	Models with Time-Varying Transition Probabilities	205
10.3.2	Endogenous Selection	208
10.3.3	Filtering and Smoothing	209

10.3.4 A Modified EM Algorithm	210
10.4 Summary and Outlook	211
11 Markov-Switching Models of the German Business Cycle	213
11.1 MS-AR Processes as Stochastic Business Cycle Models	216
11.2 Preliminary Analysis	217
11.2.1 Data	217
11.2.2 Traditional Turning Point Dating	219
11.2.3 ARMA Representation Based Model Pre-Selection	220
11.3 The Hamilton Model	222
11.3.1 Estimation Results	222
11.3.2 Contribution to the Business Cycle Characterization	224
11.3.3 Impulse Response Analysis	227
11.3.4 Asymmetries of the Business Cycle	228
11.3.5 Kernel Density Estimation	229
11.4 Models with Markov-Switching Intercepts	231
11.5 Regime-Dependent and Conditional Heteroskedasticity	235
11.6 Markov-Switching Models with Multiple Regimes	240
11.6.1 Outliers in a Three-Regime Model	240
11.6.2 Outliers and the Business Cycle	242
11.6.3 A Hidden Markov-Chain Model of the Business Cycle ..	243
11.6.4 A Highly Parameterized Model	245
11.6.5 Some Remarks on Testing	247
11.7 MS-AR Models with Regime-Dependent Autoregressive Parameters	247
11.8 An MSMH(3)-AR(4) Business Cycle Model	250
11.9 Forecasting Performance	252
11.10 Conclusions	255
11.A Appendix: Business Cycle Analysis with the Hodrick-Prescott Filter	257
12 Markov-Switching Models of Global and International Business Cycles	259
12.1 Univariate Markov-Switching Models	260
12.1.1 USA	263
12.1.2 Canada	264
12.1.3 United Kingdom	265
12.1.4 Germany	266
12.1.5 Japan	267
12.1.6 Australia	272

12.1.7 Comparisons	274
12.2 Multi-Country Growth Models with Markov-Switching Regimes ..	277
12.2.1 Common Regime Shifts in the Joint Stochastic Process of Economic Growth	277
12.2.2 Structural Breaks and the End of the Golden Age	278
12.2.3 Global Business Cycles	281
12.2.4 Rapid Growth Episodes and Recessions	284
12.3 Conclusions	288
12.A Appendix: Estimated MS-DVAR Models	290
13 Cointegration Analysis of VAR Models with Markovian Shifts in Regime	297
13.1 Cointegrated VAR Processes with Markov-Switching Regimes ...	298
13.1.1 Cointegration	298
13.1.2 The MSCI-VAR Model	299
13.1.3 A State-Space Representation for MSCI-VAR Processes ..	302
13.2 A Cointegrated VARMA Representation for MSCI-VAR Processes	306
13.3 A Two-Stage Procedure	309
13.3.1 Cointegration Analysis	309
13.3.2 EM Algorithm	311
13.4 Global and International Business Cycles	312
13.4.1 VAR Order Selection	313
13.4.2 Cointegration Analysis	314
13.4.3 Granger Causality	316
13.4.4 Forecast Error Decomposition	318
13.5 Global Business Cycles in a Cointegrated System	320
13.6 Conclusions	323
13.A Appendix: Estimated CI-VAR and MSCI-VAR Models	325
Epilogue	329
References	331
Tables	347
Figures	351
List of Notation	353