CONTENTS

PREFACE xi

Part I MATHEMATICAL PROGRAMMING 1

1 INTRODUCTION 3

1.1 Model Development 5

1.2 Early Work and Applications 8

1.3 Structural Considerations 10

1.3.1 Indifference Points and Nonexistence of Solutions 11

1.3.2 Significance of Order of Play 12

1.4 Scope and Outline 14

2 LINEAR PROGRAMMING 17

2.1 Introduction 17

2.1.1 Basic Solutions 22

2.1.2 Fundamental Theorem 23

2.1.3 Convex Properties 25

2.2 Simplex Method 29

2.2.1 Pivoting 30

2.2.2 Determining the Leaving Variable 32

2.2.3 Moving toward Optimality 33

2.2.4 Degeneracy and Cycling 37

2.3 Geometry of Simplex Method 41

2.3.1 Finiteness of Algorithm 42

2.3.2 Adjacency and Bounded Edges 42

2.3.3 Unboundedness 44

2.3.4 Finding Adjacent Extreme Points 47

2.3.5 Main Geometric Argument 47

2.3.6 Alternative Optima and Uniqueness 48

2.3.7 Ranking Extreme Points 49
2.4 Additional Features 51
2.4.1 Phase 1 and Artificial Variables 51
2.4.2 Bounded Variables 54
2.4.3 Kuhn-Tucker Conditions and the Linear Complementarity Problem 57
2.5 Duality 59
2.5.1 Primal–Dual Relationship 61
2.5.2 Dual Theorems 61
2.5.3 Economic Interpretation 66
2.5.4 Sensitivity Analysis 66
2.5.5 Dual Simplex Method 72

3 INTEGER PROGRAMMING 76
3.1 Introduction 76
3.1.1 Models with Integer Variables 78
3.1.2 Solving Integer Programs 83
3.2 Enumerative Methods 87
3.2.1 Definitions and Concepts 89
3.2.2 Generic Branch and Bound Algorithm 91
3.2.3 Branch and Bound Using LP Relaxation 92
3.2.4 Implementation Issues 96
3.2.5 Zero–One Implicit Enumeration 102
3.2.6 General Branching and Data Structures 106
3.3 Cutting Planes 109
3.3.1 Method of Integer Forms 110
3.3.2 Primal All–Integer Cuts 114
3.3.3 Cuts with Unit Coefficients 118
3.3.4 Valid Inequalities 121
3.4 Benders Decomposition for Mixed–Integer Linear Programming 127
3.4.1 Reformulation of MILP 127
3.4.2 Algorithm 130
3.5 Unimodularity 133

4 NONLINEAR PROGRAMMING 137
4.1 Introduction 137
4.1.1 Classification of Problems 140
4.1.2 Difficulties Resulting from Nonlinearities 142
4.1.3 Notation 143

4.2 Optimality Conditions 155
4.2.1 Unconstrained Problems 156
4.2.2 Nonnegative Variables 157
4.2.3 Equality Constraints 159
4.2.4 Inequality Constraints 164
4.2.5 Convex Optimization 167
4.3 Search Techniques for Unconstrained Problems 169
4.3.1 One-Dimensional Linear Search Techniques 170
4.3.2 Multidimensional Search Techniques 175
4.4 Algorithms For Constrained Optimization 181
4.4.1 Primal Methods 181
4.4.2 Penalty Methods 185
4.4.3 Sequential Quadratic Programming 188

Part II BILEVEL PROGRAMMING 193

5 LINEAR BLP: CONTINUOUS VARIABLES 195
5.1 Introduction 195
5.2 Theoretical Properties 198
5.3 Algorithms for the Linear Bilevel Programming Problem 202
5.3.1 Kth-Best Algorithm 203
5.3.2 Kuhn-Tucker Approach 204
5.3.3 Complementarity Approach 209
5.3.4 Variable Elimination Algorithm 213
5.3.5 Penalty Function Approach 218
5.4 Computational Comparisons 222

6 LINEAR BLP: DISCRETE VARIABLES 232
6.1 Introduction 232
6.2 Properties of the Zero-One Linear BLPP 233
6.2.1 Reductions to Linear Three-Level Programs 237
6.2.2 Algorithmic Implications 244
6.3 Properties of the Mixed-Integer Linear BLPP 245
6.4 Moore-Bard Algorithm for the Mixed-Integer Linear BLPP 247
6.4.1 Branch and Bound Notation 248
6.4.2 Bounding Theorems 249
6.4.3 Algorithm 250
6.4.4 Computational Experience 254
6.4.5 Assessment 257

6.5 Algorithm for the Discrete Linear BLPP 258
6.5.1 Algorithm 259
6.5.2 Computational Experience 266
6.5.3 Assessment 267

7 CONVEX BILEVEL PROGRAMMING 269
7.1 Introduction 269
7.2 Descent Approaches for the Quadratic BLPP 272
7.2.1 An *EIR* Point Descent Algorithm 274
7.2.2 A Modified Steepest Descent Approach 276
7.2.3 Hybrid Approach and Concave Minimization 283
7.3 Branch and Bound Algorithm 284
7.4 Variable Elimination Algorithm 290
7.4.1 Convex-Quadratic BLPP 291
7.4.2 Computational Experience 296

8 GENERAL BILEVEL PROGRAMMING 301
8.1 Introduction 301
8.1.1 Independence of Irrelevant Constraints 305
8.1.2 Preview of Algorithms 309
8.2 Branch and Bound Algorithm 311
8.3 Double Penalty Function Method 320
8.4 Rectangular Partitioning 327
8.5 Steepest Descent Direction 332
8.5.1 Necessary Optimality Conditions 333
8.5.2 Overview of Descent Method 335
8.6 Subgradient Descent – Bundle Method 339
8.6.1 Preliminaries 341
8.6.2 Optimality Conditions and Stability of Local Solutions 344
8.6.3 Leader Predominate Algorithm 347
8.7 Transformation to Concave Program 352
8.8 Assessment of Algorithms 359

9 HEURISTICS 361
9.1 Introduction 361
9.2 Artificial Intelligence–Based Approaches 362
9.2.1 Overview of Genetic Algorithms 363
9.2.2 GABBA 364
9.2.3 Grid Search Technique 369

9.2.4 Comparison of GABBA and Grid Search 370
9.2.5 Simulated Annealing Algorithm (SABBA) 373
9.2.6 Comparison of SABBA and Grid Search 374
9.2.7 Assessment 375
9.3 Hybrid Tabu–Descent Algorithm 375
9.3.1 Initialization Procedure 376
9.3.2 Tabu Phase 378
9.3.3 Numerical Results 381
9.3.4 Discussion 386

Part III APPLICATIONS 389

10 TRANSPORTATION NETWORK DESIGN 391
10.1 Introduction 391
10.2 Rural Highway Network 392
10.3 Decision Variables 394
10.4 BLP Formulation 395
10.5 Objective Functions 397
10.5.1 Travel Time Functions 397
10.5.2 Operating Costs 399
10.5.3 Accident Costs 402
10.5.4 Improvement and Maintenance Costs 403
10.5.5 Additivity of Cost Functions 405
10.6 Conservation of Flow Constraints 407
10.7 Solution of Empirical Problem 410
10.8 Conclusions 412

11 PRODUCTION PLANNING 414
11.1 Introduction 414
11.2 Mathematical Developments 415
11.2.1 Formulation as a Bilevel Program 415
11.2.2 Interpretation and Technical Considerations 418
11.3 Application Associated with Electric Motor Production 419
11.3.1 Solution to Continuous BLP Model 423
11.3.2 Noncooperative Implications of the Model 424
11.3.3 Solution to Mixed-Integer BLP Model 425
11.4 Discussion of Results 426

12 DETERMINING PRICE SUPPORT LEVELS FOR BIOFUEL CROPS 428

12.1 Introduction 428

12.2 Mathematical Model 430

12.3 Description of Algorithms 434

12.3.1 Industry Model 435

12.3.2 Grid Search Algorithm (GSA) 435

12.3.3 Nonlinear Programming Approach 436

12.3.4 QP Formulation for Follower's Problem 439

12.4 Implementation 440

12.4.1 Overall System Design and Components 440

12.4.2 GAMS Model Structure Determination 443

12.4.3 Model Evaluation Subsystem 446

12.5 Computational Results 449

12.5.1 Grid Search Solutions 449

12.5.2 Output from SQP 450

12.6 Discussion 452

REFERENCES 455

INDEX 469