Contents

Introduction 1

1 Linear regression models 7
1.1. Some properties of matrices 7
1.2. Linearly parametrized regression models 12
1.3. L_2 estimators of parameters 15
1.4. The Gauss-Markov theorem 23
1.5. Basic statistical properties of the estimator $\hat{\vartheta}_W$ in a regular model 25
1.6. Variance-minimizing optimal experimental designs 29

2 Linear methods in nonlinear regression models 34
2.1. Symbols for derivatives 34
2.2. Intrinsically linear regression models 36
2.3. Statistical inference in intrinsically linear models 41
2.4. Linear approximations of nonlinear regression models 43
2.5. A test of linear or intrinsically linear models against a nonlinear alternative 48
2.6. Confidence regions for ϑ obtained by linear methods 53

3 Univariate regression models 55
3.1. The model and its geometric properties 55
3.2. L_2 estimators of ϑ 61
3.3. Statistical properties of the maximum likelihood estimator 66

4 The structure of a multivariate nonlinear regression model and properties of L_2 estimators 80
4.1. Regular and singular models 80
4.2. Geometrical properties of a regular regression model — the curvatures of the model 85
4.3. Properties of singular regression models: the regression model as a differentiable manifold 95
4.3.1. The case of a singular matrix W 95
4.3.2. The case of a low rank of the matrix $J(\vartheta)$ 97
4.4. The existence and uniqueness of L_2 estimator 101

5 Nonlinear regression models: computation of estimators and curvatures 113

5.1. Iterative computation of L_2 estimators 113
5.2. The Gauss-Newton method in a regular model 117
5.3. Other methods 121
5.3.1. The gradient method 122
5.3.2. Newton's method 122
5.3.3. Quasigradient methods 123
5.3.4. The GN method when the matrix $J(\vartheta)$ is rank-deficient . 124
5.3.5. The Levenberg-Marquardt method for ill-conditioned models 124
5.4. Computation of orthonormal bases of tangent and ancillary spaces 125
5.5. Curvature arrays 129

6 Local approximations of probability densities and moments of estimators 131
6.1. Asymptotic properties of estimators: first-order local approximations 131
6.2. Second-order local approximations: approximate moments of L_2 estimators 140

7 Global approximations of densities of L_2 estimators 154
7.1. Approximate probability density of estimator $\hat{\vartheta}_W$ on the interior of the parameter space 154
7.2. Probability distribution of estimator $\hat{\vartheta}_W$ on the boundary of the parameter space 171
7.3. Probability density of the posterior modus density estimator .. 177
7.4. Probability density of $\hat{\vartheta}_C$ for $C \neq W$ 179
7.5. The probability density of the estimator when the error distribution is not normal 182
7.5.1. The case of elliptically symmetrical errors 182
7.5.2. The use of mixtures of error distributions 184
7.6. The use of the Riemannian curvature tensor in improved approximate densities 185
7.7. Conditional probability density of the estimator 188

8 Statistical consequences of global approximations especially in flat models 192
8.1. Models with a zero Riemannian curvature tensor: flat models . 192
8.2. Pivotal variables and confidence regions for ϑ in flat models ... 195
8.3. On confidence regions for ϑ in general nonlinear regression models 200
8.4. Remarks on estimators of the parameter σ in nonlinear regression 206

8.5. On optimum experimental design in nonlinear regression models 209
8.5.1. The approach based on the asymptotic normality 210
8.5.2. The sequential approach 211
8.5.3. Design of experiments based on the probability density .. 212
9 Nonlinear exponential families 215
9.1. Regular exponential families 215
9.2. Geometry of nonlinear exponential families induced by the maximum likelihood estimator 224
9.3. The saddle-point approximation of the probability density in the covering family 231
9.4. Approximate probability density of the maximum likelihood estimator in nonlinear exponential families 236
9.5. Notes on differential geometry in mathematical statistics 241
References 248
Basic symbols 255
Subject index 258