Chapter 1: Goals of Mathematics Teaching

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> 'A discussion of mathematical education, and of ways and means of enhancing its value, must be approached first of all on the basis of a precise and comprehensive formulation of the valid aims and purposes of such education. Only on such basis can we approach intelligently the problems relating to the selection and organization of material, the methods of teaching and the point of view which should govern instruction, and the qualifications and training of the teachers who impart it. Such aims and purposes of the teaching of mathematics, moreover, must be sought in the nature of the subject, the role it plays in practical, intellectual, and spiritual life of the world, and in the interests and capacities of the students.'
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> (The Reorganization of Mathematics in Secondary Education, 1923 (1970), p. 390)

ABSTRACT

This chapter attempts to analyse the justification and the goals of mathematics education from both theoretical and historical perspectives. Methodological issues related to the identification and reconstruction of the justification and the goals of mathematics education are discussed as well.

The chapter begins by setting the stage, also as far as terminology is concerned, and by asking 'what are the issues?', in order to discuss the relevance of studying the justification and goals of mathematics education. Particular emphasis is being placed on the essential distinction between descriptive/analytic and normative issues.

The main part of the chapter consists in a descriptive/analytic, internationally orientated, survey of the development of the goals of mathematics education during the past century or so, as manifested in major contributions to and documents of curriculum change as well as in contributions to the didactics of mathematics. Attempts are being made to relate the development of the goals and justification of mathematics education to the changing roles of mathematics and mathematics education in society.

The chapter concludes by placing the discourse at issue within the broader context of contemporary preoccupations and concerns in the didactics of mathematics.

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1. INTRODUCTION. WHAT ARE THE ISSUES?

1.1 Remarks on scope and terminology

In a way, most of the considerations in the present chapter pertain, I believe, to mathematics education in general, i.e. across the whole range of mathematics education from kindergarten to graduate studies. However, in order to avoid futile discussions about the extent to which the considerations are relevant to the teaching and learning of mathematics for various special categories of pupils and students, perhaps mainly at advanced levels, let me confine this chapter to dealing with the majority of pupils and students in mainstream primary and secondary education, and in tertiary education not preparing for specifically mathematical professions.

Terminological issues are mostly tedious and boring. Nevertheless, for a scholarly or scientific discourse to be serious, in fact possible, it is essential that at least the key entities and concepts of that discourse are reasonably clear to those involved. This is particularly true with a field like mathematics education in which transparency and clarity are not easily achieved, let alone to be taken as a matter of course. So, please bear with me during the few pages it takes to set the terminological stage.

Before dealing with the goals of mathematics education we need to spend a few remarks on some closely related notions which we shall also be using in this chapter: 'reason', 'justification', 'argument'. By a (real) **reason** for providing mathematics education to students within some segment of the educational system we understand a driving force, typically of a general nature, which in actual fact has motivated and given rise to the existence (i.e. the origination or the continuation) of mathematics teaching within that segment, as determined by the bodies which make the decisions (including non-decisions) in the system at issue.

Reasons for mathematics education need not be explicit, well defined and articulated, let alone agreed upon and stated in public. More often than not, reasons are implicit, indirect, fuzzy and vague, and form part of a complex conglomerate of other reasons, societal or group interests, cultural and political ideals, and so on. They include also, in fact quite frequently, impersonal societal forces, of which inertial forces are particularly relevant in this context. Only in rare cases, therefore, do we have direct access to reasons for mathematics education. For such access to be possible, it is not sufficient to identify explicitly stated and formulated reasons which seem, at first sight, to express the underlying motivation of the bodies equipped with the power to establish, continue or discontinue mathematics education. The mere fact that some explicitly stated reason for mathematics education can be identified, say in official documents, does not in itself make it a *real* reason. For this to be convincingly established, a thorough analysis of the genesis, nature, role and status of the said reason has to be carried out. In other words, it requires in-

depth research, interpretation and analysis to uncover and elucidate the real reasons of the system for establishing or maintaining mathematics education.

It further follows from the definition of 'reason', suggested above, that a reason does not have to be 'good', 'correct', 'well-founded', 'convincing', etc., relative to certain standards. What matters is that it is found to have been activated to support the existence of mathematics education. Moreover, if self-reference and circularity are to be avoided in the argument, only reasons which genuinely refer to matters outside mathematics education itself can be proper reasons. Thus, a statement like the following does not count as a proper reason: 'Our educational system should provide mathematics education to pupils and students because we think it is important to learn mathematics'. It can only become a reason if it is specified, in a non-circular way, *why* it is important to learn mathematics.

Analyses of mathematics education from historical and contemporary perspectives show that in essence there are just a few types of fundamental reasons for mathematics education. They include the following:

- contributing to the *technological and socio-economic development* of society at large, either as such or in competition with other societies/ countries;
- contributing to *society's political, ideological and cultural maintenance and development*, again either as such or in competition with other societies/countries;
- providing *individuals with prerequisites which may help them to cope with life* in the various spheres in which they live: education or occupation; private life; social life; life as a citizen.

Here, while the other two types are likely to speak for themselves, it might be worthwhile to spend a few words to explain the second type. During the last couple of centuries, societies have often seen mathematics education as contributing to the very formation of society's political, ideological and cultural 'superstructure'. Thus, in 1915 the German schoolmaster W. von Schmiedeberg (cf. Niss, 1981), won the first prize in a public competition for a treatise in which he described the ways in which mathematics education can contribute to: education for national defence; education for serious, diligent and conscientious labour; education for working in a community; and education for patriotism. He particularly insisted that mathematics education be conducive to 'absolute devotion to duty', 'subordination of the individual to the organism of work', and 'preparation for adaptation and obedience'. Soedijarto & Khodir explain (1980, p. 27) how in Indonesia in the mid-1970's mathematics education was meant to serve the general purpose of schools, which included 'to develop Indonesians who are healthy, mentally and physically, [...] are creative and responsible, are democratic and tolerant, [...] and love the nation and mankind consistent with the values of the 1945 Constitution'. Also in 1980, Kolyagin et al., in their a summary of the basic working principles for

improving mathematics syllabuses in the Soviet Union, state (p. 79) 'The school mathematics course must be directed towards the systematic inculcation in pupils of a Marxist-Leninist world outlook, [...] and towards developing pupils' cognitive independence and reasoning powers.' During the years 1988-1993, the Danish National Research Council for the Humanities financed a fairly massive research initiative under the heading 'Mathematics Education and Democracy'.

If we consider these three categories of reasons as being, so to speak, affirmative, substantive (not identical to substantiated) for the establishing of mathematics education, there are, as hinted at above, reasons which are of a different nature in that they need not be related to anything substantive, i.e. anything that pertains to mathematics education as such. Many of them are to do with tradition. Thus one reason for continuing and maintaining mathematics education might be paraphrased like this: 'Since mathematics education has been around for quite a while, it is probably good for something. Besides, they have it in all other countries too. Perhaps it would cause serious damage to our society if we reduced it or removed it from the curriculum'. Other reasons are to do with power balance and political navigation, as exemplified in the following para-statements: 'We – politicians, administrators, or institution executives – may not be convinced that mathematics education is worth its costs (in terms of economic and human resources), but riot and turmoil will break out among parents, teacher associations, employers and other sorts of lobbyists if we made drastic changes, so we had better leave it as it is.' Or: 'In face of a call for reform of our educational (sub)system, we – politicians, administrators, or institution executives – have listened with much attention to a multitude of experts, including representatives of teacher associations, researchers of mathematics education, institutions of further education, etc. This has led us to conclude that mathematics education should now be given a much stronger position in the curriculum for so and so categories of recipients.'

Whenever substantive or insubstantive reasons, of whatever nature, are activated in support of the existence of mathematics education, we shall speak about an attempt to **justify** mathematics education. Whenever reasons are put forward or invoked by participants in discussions and debates of mathematics education, we are dealing with **arguments** for mathematics education.

For a reason to make sense as such, normally it is accompanied by, or presupposes, certain corresponding explicit or implicit claims. Thus, the three fundamental substantive reasons mentioned above presuppose the following corresponding claims, respectively:

- mathematics education *can indeed* contribute to the *technological and socio-economic development* of society at large;
- mathematics education *can indeed* contribute to *society's political, ideological and cultural maintenance and development*;

– mathematics education *can indeed* contribute to providing *individuals with prerequisites which may help them to cope with life* in the various spheres in which they live.

Again, it is not essential whether a given claim is substantiated by a fair degree of evidence, or rather relies on suppositions, beliefs, or simply hopes. The important thing is that if some reason is invoked to justify mathematics education, the party invoking it has to be convinced that mathematics education can actually make a contribution as implied by the reason at issue.

It is now time to address the notion of goal. Goals of mathematics teaching and learning come in at a stage when it is given that mathematics education should exist (for whichever reasons). In what follows I shall be using the word **goal** as a comprehensive ('umbrella') term for a variety of related terms such as 'end', 'purpose', 'aim', 'objective'. These terms are supposed to be listed in increasing order of specificity and closeness. Thus, an 'end' is a final outcome – perhaps of a general, airy nature – which one intends or hopes to achieve but which may well be a point of infinity in time or space. Moreover, it is seldom clear at all how we can tell whether a given 'end' has been achieved or not. As just two examples of (different) ends we could mention 'Mathematics education should enable students to meet society's demand for a competent and flexible work force', and 'Mathematics education should enable students to master their everyday private life'. At the other extreme, 'objective' concerns fairly concrete and well defined 'here and now' matters, the achievement (or non-achievement) of which may be determined relatively easily, if not necessarily automatically. For instance, one objective could be that students should be enabled to read, interpret and judge information given in graphs and tables. In summary, a 'goal' may be anything in the spectrum from 'end' to 'objective'.

Although we have attempted, in this chapter, to make a clear distinction between 'reasons' and 'goals', by reserving the term 'reason' to concern the very existence of mathematics education vis-à-vis given categories of pupils and students and the term 'goal' to indicate the actual pursuits of mathematics teaching once it has become established, it has to be admitted that the demarcation line between the two is not always so easily drawn in practice. This is so because the goals of mathematics education are often closely related to the underlying reasons for providing it. Nevertheless, however, the relationship between reasons and goals are not deterministic, neither from a logico-philosophical nor from a pragmatic viewpoint. A given reason can give rise to, or be compatible with, several different goals. Similarly, a given goal may be pursued for several different reasons. Hence, analytically speaking, 'reasons' and 'goals' constitute relatively independent dimensions of the space we are about to explore. So, even if the title of this chapter emphasises 'goals' we shall be dealing with 'reasons' as well.