Chapter 4: Designing Curricula for Teaching and Learning Algebra

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ABSTRACT

Within this chapter we discuss what we consider to be the substantive issues related to the design and development of new algebra curricula. We focus on potential ruptures between algebra and concrete mathematical activities which are often used as a basis for teaching algebra. We emphasise the importance of mathematical sign systems for communicating and learning algebra and discuss, from this perspective, the use of computer-based environments. The main thesis of the chapter centres around the idea that algebra curricula have to take into account the teacher and teaching as well as curricular materials.

1. INTRODUCTION

Anyone involved in the development of a new algebra curriculum will inevitably have a conscious or unconscious view of what mathematics and more specifically what algebra is. This will be reflected in the type of curriculum which is produced, which in its turn will be reflected in the different practices in the classroom. Although this chapter is concerned with designing curricula for teaching and learning algebra, views about algebra cannot be separated from views about mathematics. For this reason the first section of the chapter is more generally concerned with mathematics, and the issue of algebra emerges from this general discussion.

We do not intend here, to enter into a discussion of what mathematics is; instead, we present a set of views about school mathematics, which although not exhaustive, provide a starting point for our discussion. Our view is that it is important to develop a balanced curriculum. If some of the perspectives presented in Table 1 are given preference over others, then the resulting bias reduces the possibility of using the curriculum to achieve a rich and novel learning experience for pupils. A false disjunction arises, for example when the relational aspect of mathematical thinking detracts from its instrumental use and vice versa. Another example is when the problem-solving aspect of mathematics is falsely separated from mathematical knowledge. Within a bi-

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ased curriculum the tension between comprehension and mere mechanisation is likely to become more acute. Consider, for example, an approach which assumes that school mathematics is knowledge about given (ideal) objects, whose properties and relationships should be discovered. This perspective is in opposition to other radical tendencies which maintain that all mathematical knowledge is constructed from the first interactions between a subject and the real world. Both perspectives ignore all the social aspects which intervene in the processes by which students become competent in, for example, the use of mathematical language in order to think and also produce practical knowledge which can be communicated to others.

However, in contrast to the partial and biased view of the curriculum, perhaps the most common mistake is to fall into a complete eclecticism, trying to give the same weight to most of the aspects listed in Table 1. This usually leads to curriculum design in which confusion reaches the most elementary parts of what is produced and in which the design is a mass of contradictions.

In view of the above discussion, we suggest that it is important to begin curricular design with a 'good enough' general framework which is based on some clearly established points of view, which can be refined as the curriculum is developed. This framework will provide some principles from which the developing curriculum can emerge. In this way there is an attempt to dilute the tensions between the different perspectives about what mathematics is, because of a need to respond to the demands (in terms of curricular implementation) of the chosen framework. These become converted into 'lines of force' which give impulse to some decisions and not to others and, more importantly, give meaning to such decisions. We suggest that the following six criteria can act as a framework from which all aspects of algebra curriculum development can be analysed.

- Mathematical concepts and related sign systems,
- Mathematical cognition and cognitive tendencies,
- Teaching and learning and abstraction processes,
- The relationship between algebra and practical knowledge,
- New technologies for teaching and learning algebra,
- Mathematical modelling—the analytical and instrumental tool of algebra within other areas of knowledge

School mathematics can be referred to as:
– A body of knowledge to be learned.
– A set of techniques for solving problems.
– The study of certain structures: the arithmetic-algebraic, the geometric, etc.

Table 1: Views about school mathematics.

School mathematics can be referred to as:
– A language with a given system of signs which is interwoven with natural language.
– A formal science with a highly formalised language.
– A scientific activity, that of mathematicians, which is centuries old and which in this century has developed specific practices very far removed from those found in educational systems.
– An activity in which phenomena pertaining to the natural and social sciences are modelled.
– A collection of procedures for carrying out practical calculations in order to measure classify, predict, count, etc.
– A part of natural language in which judgements as to the progress of society, the economy, the climate, the expectations of voting behaviour, etc. are expressed.
– A collection of ways of speaking about chance and repetitive phenomena in order to predict certain future events.
– An essential element of the culture of all times.
– A symbolic system in which expressions can be formulated that give account of general patterns and thus generalised calculations can be carried out.
– A symbolic system in which generalisations and abstractions can be expressed.
– A symbolic system in which phenomena of iteration and recurrence can be used to express algorithms.
– A system of mental abilities such as spatial imagination, the capacity of hypothetical and deductive reasoning, etc.
– Structures of the intellect, internalisation of the properties of actions carried out with real objects.
– A list (even longer than the preceding one) of the activities of teaching such as that found in textbooks on the subject etc.

Table 1: Views about school mathematics.

2. MATHEMATICAL CONCEPTS AND RELATED SIGN SYSTEMS

School mathematics is articulated as a series of interrelated conceptual networks, with the characteristic that, with time, students achieve competence in the use of increasingly more sophisticated and elaborated networks of concepts. Although there is no absolute hierarchy within concept development certain competencies require many other competencies to have been previously mastered.

Whereas the partial stratification of mathematical concepts is well accepted, a similar stratification of mathematical sign systems (e.g., graphs, algebra code, diagrams, tally systems) in which concepts are expressed and communicated has received less attention. This stratification relates to the diversity of mathematical objects, actions, operations and transformations of these objects, whose generality and abstractness depends on the sign systems being used. We maintain that curriculum designers should have a sufficiently broad concept of mathematical sign systems[1] (referred to as MSS) and a notion of the meaning of these sign systems which embraces both a more formal mathematical meaning and a more pragmatic one.

Many curriculum developers have tended to ignore the importance of mathematical sign systems, and have viewed these as merely 'add ons' to the conceptualisation process. However, mathematics cannot be communicated without the use of sign systems. In many situations teachers and students will work with intermediate sign systems (for example balances, piles of rocks, diagrams, etc.). These intermediate sign systems are either introduced by the teacher or textbook or constructed by the learner. They will ultimately become modified so that, at the end of the teaching/learning process, the student becomes competent with the appropriate conventional mathematical sign systems. As Kaput (1989) has pointed out, a theory for the production of mathematical sign systems has to deal with at least four sources of meaning resulting from:

1) Transformation within a particular mathematical sign system without reference to another system (for example, when $F = \frac{9}{5}C + 32$ is transformed to $C = \frac{5(F-32)}{9}$.
2) Translation across mathematical sign systems (for example, when $F = \frac{9}{5}C + 32$ is transformed to a straight line graph of temperature in degrees Fahrenheit against temperature in degrees centigrade).
3) Translations between mathematical sign systems and nonmathematical sign systems, such as natural language, visual images and the behavioural sign systems used by students during the learning/teaching processes (for example, when the student uses gesture or natural language).
4) Consolidation, simplification, generalisation and reification of actions, procedures and concepts of the intermediate mathematical sign systems created during development of the teaching sequences proposed by the teaching model being used. These intermediate mathematical sign systems evolve into a new, 'more abstract' MSS in which there will be new actions, procedures and concepts that will have as referents all the relevant actions, procedures and concepts of the intermediate MSS for

> their use in new signification processes. If the goals of teaching and learning are achieved, the new stage has a higher level of organisation and represents a corresponding new departure point in the cognitive development of the learner.

While the first three sources of meaning represent ways of dealing with primitive expressions and means of combining them, the fourth represents means of abstraction, by which compound objects can be named and manipulated as units and afterwards used in signification processes to solve new problem-solving situations which are presented to the learner in the sequence of teaching activities. What we use in order to think mathematically and to communicate our thoughts to others is a *collection of stratified mathematical sign systems* with interrelated codes that allow the production of texts whose decoding will have to refer to several of those strata.

The main goal of a mathematics curriculum is that the learner will become a competent user of mathematics and mathematical sign systems (e.g., the algebra code). Taking into account the learner implies taking into account the cognitive processes involved. All teaching/learning situations are potential sources of new concepts. Even when small changes are made to the mathematical sign system in which 'old knowledge' is being described, the students are constructing new conceptions. This idea stands in marked contrast to what mathematicians (who are mostly Platonists) argue when they maintain that changing a sign system does not change a pre-existing notion which has already been represented by another sign system.

The idea that changing the sign system does not change the concept is also implicit in many existing mathematical texts. In fact, the notion of mathematical understanding is often predicated on the view that understanding comes before, and is thus separated from external representation. We, however, take the Vygotskian perspective in which the sign systems which derive from culture (in this case mathematics culture) take on firstly a social and communicative function, and only later become internalised, developing into a new tool to think with. Inducting students into the communicative function of mathematical sign systems has to be done by the teacher, and here the teacher is leading the students ahead of their development. In other words, they will be inevitably using sign systems without fully understanding what they are doing, as is the case when a baby learns to talk.

> One of the correlates of the fact that interpsychological semiotic processes require the use of external sign forms is that it is possible to produce such forms without recognising the full significance that is attached to them by others. As a result it is possible for a child to use seemingly appropriate communication behaviour before recognising all aspects of its significance as understood by more experienced members of the culture. One of the mechanisms that makes possible the cognitive