

# Contents

<b>1. Weak Convergence of Stochastic Processes</b> .....	<b>1</b>
1.1 Basic Properties of Stochastic Processes .....	2
1.1.1 Stochastic Basis, Filtration, Stopping Times .....	2
1.1.2 Stochastic Processes .....	3
1.1.3 Martingales .....	7
1.1.4 Semimartingales and Stochastic Integrals .....	14
1.1.5 Markov Processes and Stochastic Differential Equations	40
1.1.6 The Discrete Time Case .....	58
1.2 Weak Convergence .....	64
1.2.1 The Skorokhod Topology .....	64
1.2.2 Continuity for the Skorokhod Topology .....	67
1.2.3 Definition of Weak Convergence .....	69
1.2.4 Criteria for Tightness in $\mathbb{D}^k$ .....	72
1.2.5 The Meyer-Zheng Topology .....	74
1.3 Weak Convergence to a Semimartingale .....	75
1.3.1 Functional Convergence and Characteristics .....	75
1.3.2 Limits of Martingales .....	87
1.3.3 Limit Theorems for Markov Processes .....	88
1.3.4 Convergence of Triangular Arrays .....	92
1.4 Weak Convergence of Stochastic Integrals .....	100
1.4.1 Introduction .....	100
1.4.2 The Uniform Tightness Condition U.T. ....	101
1.4.3 Functional Limit Theorems for Sequences of Stochastic Integrals and Stochastic Differential Equations.....	104
1.5 Limit Theorems, Density Processes and Contiguity .....	108
1.5.1 Hellinger Integral and Hellinger Process .....	108
1.5.2 Contiguity and Entire Separation .....	115
1.5.3 Convergence of the Density Processes .....	121
1.5.4 The Statistical Invariance Principle .....	125

<b>2. Weak Convergence of Financial Markets</b> .....	129
2.1 Convergence of Optimal Consumption-Portfolio Strategies ...	130
2.1.1 Weak Convergence of Controlled Processes .....	131
2.1.2 The Martingale Approach .....	161
2.2 Convergence of Options Prices .....	185
2.2.1 Problems and Examples .....	185
2.2.2 Contiguity Properties .....	194
2.2.3 The Case of Incomplete Markets .....	198
2.2.4 Transaction Costs .....	218
2.2.5 American Options .....	230
2.3 Convergence of Hedging Strategies .....	240
2.3.1 Binomial Case and Clark-Haussman Formula .....	243
2.3.2 Weak Convergence of Integrands .....	251
2.3.3 The Local Risk-Minimizing Strategy .....	256
<b>3. The Basic Models of Approximations</b> .....	267
3.1 General Remarks .....	267
3.1.1 Some numerical methods for forward and backward stochastic differential equations .....	268
3.1.2 Some numerical methods for computations of Greeks ..	272
3.2 Lattice .....	274
3.2.1 Simple Binomial Processes as Diffusion Approximations	274
3.2.2 Correction Terms for Path-Dependent Options .....	287
3.2.3 Adjustment Prior to Maturity and Smoothing of the Payoff Functions .....	295
3.2.4 Fast Accurate Binomial Pricing .....	301
3.2.5 Approximating a Diffusion by a Trinomial Tree .....	305
3.3 Alternative Approximations .....	309
3.3.1 ARCH Approximations .....	309
3.3.2 Lévy Processes .....	321
3.3.3 Convergence for Random Time Intervals .....	345
3.3.4 Deterministic or Random Discretizations of Continuous- Time Processes .....	359
3.4 Approximations of Term Structure Models .....	371
3.4.1 Bonds and Interest Rate Derivatives .....	371
3.4.2 Basic Interest Models and their Approximations .....	377
3.4.3 Two-factors Model .....	387
3.4.4 Market Models : Discretization of Lognormal Forward Libor and Swap Rate Models .....	388
3.4.5 Discretization of Deflated Bond Prices .....	388
3.4.6 Pricing Interest Rate or Equity Derivatives and Dis- cretization .....	395
<b>Index</b> .....	419