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Preface

The purpose of this article is to study in detail the actions of a semisimple Lie or algebraic group on its Lie algebra by the adjoint representation and on itself by the adjoint action. We will focus primarily on orbits through nilpotent elements in the Lie algebra; these are called nilpotent orbits for short. Many deep results about such orbits have been obtained in the last thirty-five years; we will collect some of the most significant of these that have found wide application to representation theory. We will primarily work in the setting of a semisimple Lie algebra and its adjoint group over an algebraically closed field of characteristic zero, but we will extend much of what we do to semisimple Lie algebras over the reals or an algebraically closed field of prime characteristic, and to conjugacy classes in semisimple algebraic groups. We will give detailed proofs of many results, including some which are difficult to ferret out of the literature. Other results will be summarized with reasonably complete references. The treatment is a more comprehensive version of that in [CM93]; there is also some overlap with Humphreys's book [Hu95]. In the last chapter we summarize some of the most recent work being done in this topic and indicate some directions of current research.

The reader is expected to be familiar with the structure and classification of complex semisimple Lie algebras, together with the basic definitions and theorems typically found in a first course on that subject. We will also invoke the corresponding facts about real Lie groups and algebras and algebraic groups from time to time. For convenience this background material is summarized in Chapter 1. The classical matrix groups and algebras will serve as a ready source of examples; we will often be able to derive very explicit results for such groups and algebras using nothing more than linear algebra.