

# Contents

Preface .....	V
Contents .....	IX
Notation .....	XIII
<b>Chapter 1 Modeling Tools for Financial Options .....</b>	<b>1</b>
1.1 Options .....	1
1.2 Model of the Financial Market .....	7
1.3 Numerical Methods .....	10
1.4 The Binomial Method .....	12
1.5 Risk-Neutral Valuation .....	21
1.6 Stochastic Processes .....	24
1.6.1 Wiener Process .....	26
1.6.2 Stochastic Integral .....	28
1.7 Stochastic Differential Equations .....	31
1.7.1 Itô Process .....	31
1.7.2 Application to the Stock Market .....	34
1.8 Itô Lemma and Implications .....	38
Notes and Comments .....	41
Exercises .....	45
<b>Chapter 2 Generating Random Numbers with Specified Distributions .....</b>	<b>51</b>
2.1 Pseudo-Random Numbers .....	51
2.1.1 Linear Congruential Generators .....	52
2.1.2 Random Vectors .....	53
2.1.3 Fibonacci Generators .....	56
2.2 Transformed Random Variables .....	57
2.2.1 Inversion .....	58
2.2.2 Transformation in $\mathbb{R}^1$ .....	60
2.2.3 Transformation in $\mathbb{R}^n$ .....	61
2.3 Normally Distributed Random Variables .....	62
2.3.1 Method of Box and Muller .....	62
2.3.2 Method of Marsaglia .....	63
2.3.3 Correlated Random Variables .....	64

2.4 Sequences of Numbers with Low Discrepancy .....	66
2.4.1 Monte Carlo Integration .....	66
2.4.2 Discrepancy .....	67
2.4.3 Examples of Low-Discrepancy Sequences .....	70
Notes and Comments .....	72
Exercises .....	74
<b>Chapter 3 Numerical Integration of Stochastic Differential Equations .....</b>	<b>79</b>
3.1 Approximation Error .....	80
3.2 Stochastic Taylor Expansion .....	83
3.3 Examples of Numerical Methods .....	86
3.4 Intermediate Values .....	89
3.5 Monte Carlo Simulation .....	90
3.5.1 The Basic Version .....	90
3.5.2 Variance Reduction .....	92
Notes and Comments .....	95
Exercises .....	97
<b>Chapter 4 Finite Differences and Standard Options .....</b>	<b>99</b>
4.1 Preparations .....	100
4.2 Foundations of Finite-Difference Methods .....	102
4.2.1 Difference Approximation .....	102
4.2.2 The Grid .....	103
4.2.3 Explicit Method .....	104
4.2.4 Stability .....	106
4.2.5 Implicit Method .....	109
4.3 Crank-Nicolson Method .....	110
4.4 Boundary Conditions .....	113
4.5 American Options as Free Boundary-Value Problems .....	116
4.5.1 Free Boundary-Value Problems .....	116
4.5.2 Black-Scholes Inequality .....	120
4.5.3 Obstacle Problems .....	120
4.5.4 Linear Complementarity for American Put Options ..	123
4.6 Computation of American Options .....	124
4.6.1 Discretization with Finite Differences .....	125
4.6.2 Iterative Solution .....	126
4.6.3 Algorithm for Calculating American Options .....	128
4.7 On the Accuracy .....	132
Notes and Comments .....	136
Exercises .....	138

<b>Chapter 5 Finite-Element Methods .....</b>	141
5.1 Weighted Residuals .....	142
5.1.1 The Principle of Weighted Residuals .....	143
5.1.2 Examples of Weighting Functions .....	144
5.1.3 Examples of Basis Functions .....	145
5.2 Galerkin Approach with Hat Functions .....	146
5.2.1 Hat Functions .....	147
5.2.2 A Simple Application .....	149
5.3 Application to Standard Options .....	152
5.4 Error Estimates .....	156
5.4.1 Classical and Weak Solutions .....	156
5.4.2 Approximation on Finite-Dimensional Subspaces .....	158
5.4.3 Céa's Lemma .....	160
Notes and Comments .....	162
Exercises .....	163
<b>Chapter 6 Pricing of Exotic Options .....</b>	165
6.1 Exotic Options .....	166
6.2 Asian Options .....	168
6.2.1 The Payoff .....	168
6.2.2 Modeling in the Black-Scholes Framework .....	169
6.2.3 Reduction to a One-Dimensional Equation .....	170
6.2.4 Discrete Monitoring .....	172
6.3 Numerical Aspects .....	173
6.3.1 Convection-Diffusion Problems .....	174
6.3.2 Von Neumann Stability Analysis .....	177
6.4 Upwind Schemes and Other Methods .....	178
6.4.1 Upwind Scheme .....	179
6.4.2 Dispersion .....	180
6.5 High-Resolution Methods .....	183
6.5.1 The Lax-Wendroff Method .....	183
6.5.2 Total Variation Diminishing .....	184
6.5.3 Numerical Dissipation .....	185
Notes and Comments .....	187
Exercises .....	188
<b>Appendices .....</b>	191
A1 Financial Derivatives .....	191
A2 Essentials of Stochastics .....	194
A3 The Black-Scholes Equation .....	197
A4 Numerical Methods .....	200
A5 Iterative Methods for $Ax = b$ .....	204
A6 Function Spaces .....	206
A7 Complementary Formula .....	209

XII      Contents

<b>References</b> .....	211
<b>Index</b> .....	219