15 The Baryons

The best known baryons are the proton and the neutron. These are collectively referred to as the nucleons. Our study of deep inelastic scattering has taught us that they are composed of three valence quarks, gluons and a "sea" of quark-antiquark pairs. The following treatment of the baryonic spectrum will, analogously to our description of the mesons, be centred around the concept of the constituent quark.

Nomenclature. This chapter will be solely concerned with those baryons which are made up of u-, d- and s-quarks. The baryons whose valence quarks are just u- and d-quarks are the nucleons (isospin $I=1/2$) and the Δ particles ($I=3/2$). Baryons containing s-quarks are collectively known as *hyperons*. These particles, the Λ, Σ, Ξ and Ω, are distinguished from each other by their isospin and the number of s-quarks they contain.

Name		N	Δ	Λ	Σ	Ξ	Ω
Isospin	I	1/2	3/2	0	1	1/2	0
Strangeness	S	0		−1		−2	−3
Number of s-quarks		0		1		2	3

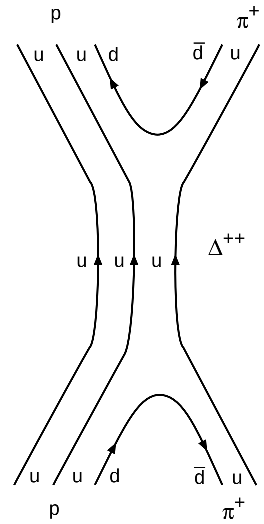
The antihyperons have strangeness +1, +2 or +3 respectively.

The discovery of baryons containing c- and b-quarks has caused this scheme to be extended. The presence of quarks heavier than the s is signified by an subscript attached to the relevant hyperon symbol: thus the Λ_c^+ corresponds to a (udc) state and the Ξ_{cc}^{++} has the valence structure (ucc). Such heavy baryons will not, however, be handled in what follows.

15.1 The Production and Detection of Baryons

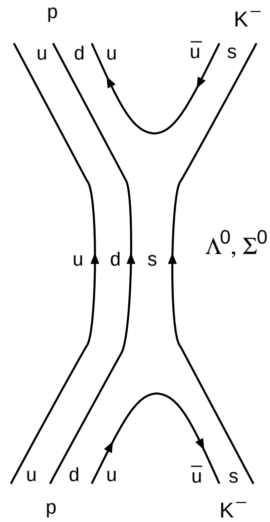
Formation experiments. Baryons can be produced in many different ways in accelerators. In Sect. 7.1 we have already described how nucleon resonances may be produced in inelastic electron scattering. These excited nucleon states are also created when pions are scattered off protons.

One can then study, for example, the energy (mass) and width (lifetime) of the Δ^{++} resonance in the reaction



$$\pi^+ + \mathrm{p} \rightarrow \Delta^{++} \rightarrow \mathrm{p} + \pi^+$$

by varying the energy of the incoming pion beam and measuring the total cross section. The largest and lowest energy peak in the cross section is found at 1232 MeV. This is known as the $\Delta^{++}(1232)$. The diagram shows its creation and decay in terms of quark lines. In simple terms we may say that the energy which is released in the quark-antiquark annihilation is converted into the excitation energy of the resonance and that this process is reversed in the decay of the resonance to form a new quark-antiquark pair. This short lived state decays about $0.5 \cdot 10^{-23}$ s after it is formed and it is thus only possible to detect the decay products, i.e., the proton and the π^+. Their angular distribution, however, may be used to determine the resonances' spin and parity. The result is found to be $J^P = 3/2^+$. The extremely short lifetime attests to the decay taking place through the strong interaction. At higher centre of mass energies in this reaction further resonances may be seen in the cross section. These correspond to excited Δ^{++} states where the quarks occupy higher energy levels. Strangeness may be brought into the game by replacing the pion beam by a kaon beam and one may thus generate hyperons. A possible reaction is



$$\mathrm{K}^- + \mathrm{p} \rightarrow \Sigma^{*0} \rightarrow \mathrm{p} + \mathrm{K}^- \ .$$

The intermediate resonance state, an excited state of the Σ^0, is, like the Δ^{++}, extremely short lived and "immediately" decays, primarily back into a proton and a negatively charged kaon. The quark line diagram offers a general description of all those resonances whose quark composition is such that they may be produced in this process. Thus excited Λ^0's may also be created in the above reaction. The cross sections of the above reactions are displayed in Fig. 15.1 as functions of the centre of mass energy. The resonance structures may be easily recognised. The individual peaks, which give us the masses of the excited baryon states, are generally difficult to separate from each other. This is because their widths are typically of the order of 100 MeV and the various peaks hence overlap. Such large widths are characteristic for particles which decay via strong processes.

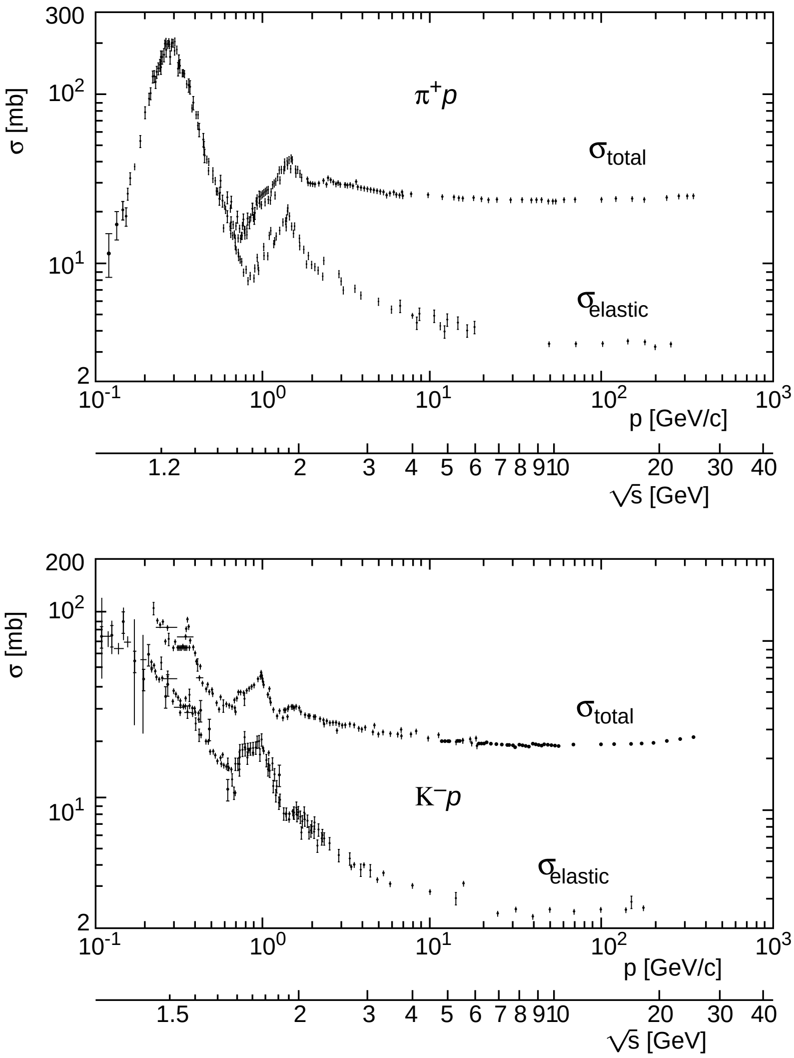


Fig. 15.1. The total and elastic cross sections for the scattering of π^+ mesons off protons (*top*) and of K^- mesons off protons (*bottom*) as a function of the mesonic beam energy (or centre of mass energy) [PD98]. The peaks are associated with short lived states, and since the total initial charge in π^+p scattering is $+2e$ the relevant peaks must correspond to the Δ^{++} particle. The strongest peak, at a beam energy of around 300 MeV/c is due to the ground state of the Δ^{++} which has a mass of 1232 MeV/c^2. The resonances that show up as peaks in the K^-p cross section are excited, neutral Σ and Λ baryons. The most prominent peaks are the excited $\Sigma^0(1775)$ und $\Lambda^0(1820)$ states which overlap significantly.

In *formation experiments*, like those treated above, the baryon which is formed is detected as a resonance in a cross section. Due to the limited number of particle beams available to us this method may only be used to generate nucleons and their excited states or those hyperons with strangeness $S = -1$.

Production experiments. A more general way of generating baryons is in *production experiments*. In these one fires a beam of protons, pions or kaons with as high an energy as possible at a target. The limit on the energy available for the production of new particles is the centre of mass energy of the scattering process. As can be seen from Fig. 15.1, for centre of mass energies greater than 3 GeV no further resonances can be recognised and the elastic cross section is thereafter only a minor part of the total cross section. This energy range is dominated by inelastic particle production.

In such production experiments one does not look for resonances in the cross section but rather studies the particles which are created, generally in generous quantities, in the reactions. If these particles are short lived, then it is only possible to actually detect their decay products. The short lived states can, however, often be reconstructed by the invariant mass method. If the momenta $\boldsymbol{p}_i$ and energies E_i of the various products can be measured, then we may use the fact that the mass M_{X} of the decayed particle X is given by

$$M_{\mathrm{X}}^2 c^4 = p_{\mathrm{X}}^2 c^2 = \left(\sum_i p_i c\right)^2 = \left(\sum_i E_i\right)^2 - \left(\sum_i \boldsymbol{p}_i c\right)^2 . \tag{15.1}$$

In practice one studies a great number of scattering events and calculates the invariant mass of some particular combination of the particles which have been detected. Short lived resonances which have decayed into these particles reveal themselves as peaks in the invariant mass spectrum. On the one hand we may identify short lived resonances that we already knew about in this way, on the other hand we can thus see if new, previously unknown particles are being formed.

As an example consider the invariant mass spectrum of the $\Lambda^0 + \pi^+$ final particles in the reaction

$$\mathrm{K}^- + \mathrm{p} \to \pi^+ + \pi^- + \Lambda^0 .$$

This displays a clear peak at 1385 MeV/c^2 (Fig. 15.2) which corresponds to an excited Σ^+. The Σ^{*+} baryon is therefore identified from its decay into $\Sigma^{*+} \to \pi^+ + \Lambda^0$. Since this is a strong decay all quantum numbers, e.g., strangeness and isospin, are conserved. In the above reaction it is just as likely to be the case that a Σ^{*-} state is produced. This would then decay into $\Lambda^0 + \pi^-$. Study of the invariant masses yields almost identical masses for these two baryons.[1] This may also be read off from Fig. 15.2. The somewhat

[1] The mass difference between the Σ^{*-} and the Σ^{*+} is roughly 4 MeV/c^2 (see Table 15.1 on p. 210).

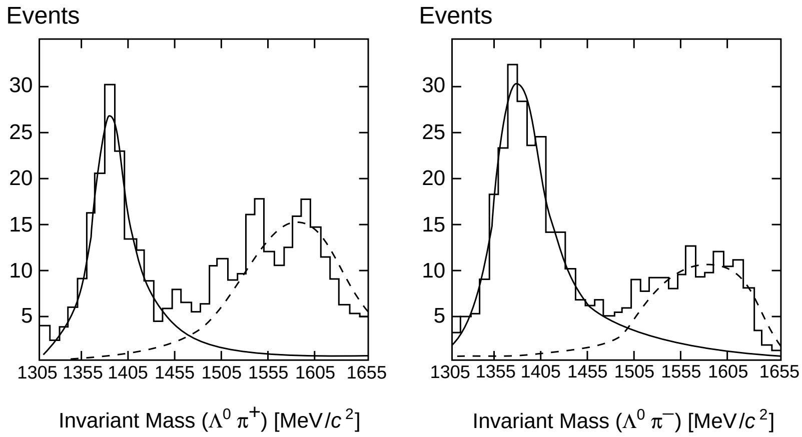


Fig. 15.2. Invariant mass spectrum of the particle combinations $\Lambda^0+\pi^+$ (*left*) and $\Lambda^0+\pi^-$ (*right*) in the reaction $K^-+p \to \pi^++\pi^-+\Lambda^0$. The momentum of the initial kaon was 1.11 GeV/c. The events were recorded in a bubble chamber. Both spectra display a peak around 1385 MeV/c^2, which correspond to Σ^{*+} and Σ^{*-} respectively. A Breit-Wigner distribution (*continuous line*) has been fitted to the peak. The mass and width of the resonance may be found in this way. The energy of the pion which is not involved in the decay is kinematically fixed for any particular beam energy. Its combination together with the Λ^0 yields a "false" peak at higher energies which does not correspond to a resonance (from [El61]).

flatter peak at higher energies visible in both spectra is a consequence of the possibility to create either of these two charged Σ resonances: the momentum and energy of the pion which is not created in the decay is fixed and so creates a "'fictitious"' peak in the invariant mass spectrum. This ambiguity can be resolved by carrying out the experiment at differing beam energies. There is a further small background in the invariant mass spectrum which is not correlated with the above, i.e., it does not come from $\Sigma^{*\pm}$ decay. We note that the excited Σ state was first found in 1960 using the invariant mass method [Al60].

If the baryonic state that we wish to investigate is already known, then the resonance may be investigated in individual events as well. This is, for example, important for the above identification of the Σ^{*+}, since the Λ^0 itself decays via $\Lambda^0 \to p+\pi^-$ and must first be reconstructed by the invariant mass method. The detection of the Λ^0 is rendered easier by its long lifetime of $2.6\cdot10^{-10}$ s (due to its weak decay). On average the Λ^0 transverses a distance from several centimetres to a few metres, this depends upon its energy, before it decays. From the tracks of its decay products the position of the Λ^0's decay may be localised and distinguished from that of the primary reaction.

A nice example of such a step by step reconstruction of the initially created, primary particles from a Σ^-+nucleus reaction is shown in Fig. 15.3.

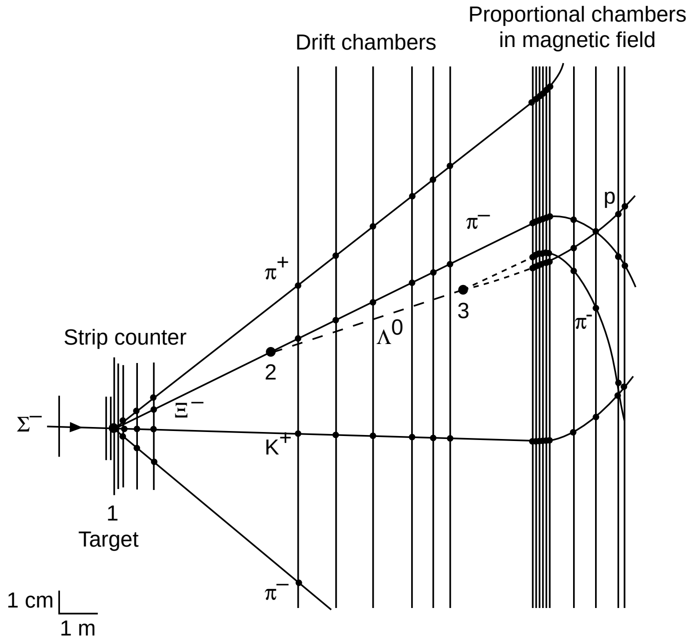


Fig. 15.3. Detection of a baryon decay cascade at the WA89 detector at the CERN hyperon beam (based upon [Tr92]). In this event a Σ^- hyperon with 370 GeV kinetic energy hits a thin carbon target. The paths of the charged particles thus produced are detected near to the target by silicon strip detectors and further away by drift and proportional chambers. Their momenta are determined by measuring the deflection of the tracks in a strong magnetic field. The tracks marked in the figure are based upon the signals from the various detectors. The baryonic decay chain is described in the text.

The method of invariant masses could be used to show a three step process of baryon decays. The measured reaction is

$$\Sigma^- + \mathrm{A} \to \mathrm{p} + \mathrm{K}^+ + \pi^+ + \pi^- + \pi^- + \pi^- + \mathrm{A}' .$$

The initial reaction takes place at one of the protons of a nucleus A. All of the particles in the final state were identified (except for the final nucleus A′) and their momenta were measured. The tracks of a proton and a π^- could be measured in drift and proportional chambers and followed back to the point (3), where a Λ^0 decayed (as a calculation of the invariant mass of the proton and the π^- shows). Since we thus have the momentum of the Λ^0 we can extrapolate its path back to (2) where it meets the path of a π^-. The invariant mass of the Λ^0 and of this π^- is roughly 1320 MeV/c^2 which is the mass of the Ξ^- baryon. This baryon can in turn be traced to the target at (1). The analysis then shows that the Ξ^- was in fact the decay product of a primary Ξ^{*0} state which "instantaneously" decayed via the strong interaction into a Ξ^- and a π^+. The complete reaction in all its glory was therefore the following

$$\begin{aligned}
\Sigma^- + \mathrm{A} \to\ & \Xi^{*0} + \mathrm{K}^+ + \pi^- + \mathrm{A}' \\
& \hookrightarrow \Xi^- + \pi^+ \\
& \qquad \hookrightarrow \Lambda^0 + \pi^- \\
& \qquad\qquad \hookrightarrow \mathrm{p} + \pi^- \,.
\end{aligned}$$

This reaction also exemplifies the associated production of strange particles: the Σ^- from the beam had strangeness -1 and yet produces in the collision with the target a Ξ^{*0} with strangeness -2. Since the strange quantum number is conserved in strong interactions an additional K^+ with strangeness $+1$ was also created.

15.2 Baryon Multiplets

We now want to describe in somewhat more detail which baryons may be built up from the u-, d- and s-quarks. We will though limit ourselves to the lightest states, i.e., those where the quarks have relative orbital angular momentum $\ell = 0$ and are not radially excited.

The three valence quarks in the baryon must, by virtue of their fermionic character, satisfy the Pauli principle. The total baryonic wavefunction

$$\psi_{\text{total}} = \xi_{\text{spatial}} \cdot \zeta_{\text{flavour}} \cdot \chi_{\text{spin}} \cdot \phi_{\text{colour}}$$

must in other words be antisymmetric under the exchange of any two of the quarks. The total baryonic spin S results from adding the three individual quark spins ($s = 1/2$) and must be either $S = 1/2$ or $S = 3/2$. Since we demand that $\ell = 0$, the total angular momentum J of the baryon is just the total spin of the three quarks.

The baryon decuplet. Let us first investigate the $J^P = 3/2^+$ baryons. Here the three quarks have parallel spins and the spin wave function is therefore symmetric under an interchange of two of the quarks. For $\ell = 0$ states this is also true of the spatial wave function. Taking, for example, the uuu state it is obvious that the flavour wave function has to be symmetric and this then implies that the colour wave function must be totally antisymmetric in order to yield an antisymmetric total wave function and so fulfill the Pauli principle. Because baryons are colourless objects the totally antisymmetric colour wave function can be constructed as follows:

$$\phi_{\text{colour}} = \frac{1}{\sqrt{6}} \sum_{\alpha=r,g,b} \sum_{\beta=r,g,b} \sum_{\gamma=r,g,b} \varepsilon_{\alpha\beta\gamma} \, |\mathrm{q}_\alpha \mathrm{q}_\beta \mathrm{q}_\gamma\rangle \,, \tag{15.2}$$

where we sum over the three colours, here denoted by *red, green* and *blue*, and $\varepsilon_{\alpha\beta\gamma}$ is the totally antisymmetric tensor.

If we do not concern ourselves with radial excitations, we are left with ten different systems that can be built out of three quarks, are $J^P = 3/2^+$ and have totally antisymmetric wave functions. These are

$$|\Delta^{++}\rangle = |u^\uparrow u^\uparrow u^\uparrow\rangle \quad |\Delta^{+}\rangle = |u^\uparrow u^\uparrow d^\uparrow\rangle \quad |\Delta^{0}\rangle = |u^\uparrow d^\uparrow d^\uparrow\rangle \quad |\Delta^{-}\rangle = |d^\uparrow d^\uparrow d^\uparrow\rangle$$

$$|\Sigma^{*+}\rangle = |u^\uparrow u^\uparrow s^\uparrow\rangle \qquad |\Sigma^{*0}\rangle = |u^\uparrow d^\uparrow s^\uparrow\rangle \qquad |\Sigma^{*-}\rangle = |d^\uparrow d^\uparrow s^\uparrow\rangle$$

$$|\Xi^{*0}\rangle = |u^\uparrow s^\uparrow s^\uparrow\rangle \quad |\Xi^{*-}\rangle = |d^\uparrow s^\uparrow s^\uparrow\rangle$$

$$|\Omega^{-}\rangle = |s^\uparrow s^\uparrow s^\uparrow\rangle \; .$$

Note that we have only given the spin-flavour part of the total baryonic wave function here, and that in an abbreviated fashion. It must be symmetric under quark exchange. In the above notation this is evident for the pure uuu, ddd and sss systems. For baryons built out of more than one quark flavour the symmetrised version contains several terms. Thus then the symmetrised part of the wave function of, for example, the Δ^+ reads more fully:

$$|\Delta^+\rangle = \frac{1}{\sqrt{3}} \left\{ |u^\uparrow u^\uparrow d^\uparrow\rangle + |u^\uparrow d^\uparrow u^\uparrow\rangle + |d^\uparrow u^\uparrow u^\uparrow\rangle \right\} \; .$$

In what follows we will mostly employ the abbreviated notation for the baryonic quark wave function and quietly assume that the total wave function has in fact been correctly antisymmetrised.

If we display the states of this baryon decuplet on an I_3 vs. S plot, we obtain (Fig. 15.4) an isosceles triangle. This reflects the threefold symmetry of these three-quark systems.

The baryon octet. We are now faced with the question of bringing the nucleons into our model of the baryons. If three quarks, each with spin 1/2, are to yield a spin 1/2 baryon, then the spin of one of the quarks must be antiparallel to the other two, i.e., we must have $\uparrow\uparrow\downarrow$. This spin state is then neither symmetric nor antisymmetric under spin swaps, but rather has a mixed symmetry. This must then also be the case for the flavour wave function, so that their product, the total spin-flavour wave function, is purely symmetric. This is not possible for the uuu, ddd and sss quark combinations and indeed we do not find any ground state baryons of this form with $J = 1/2$. There are then only two different possible combinations of u and d quarks which can fulfill the necessary symmetry conditions on the wave function of a spin 1/2 baryon, and these are just the proton and the neutron.

This simplified treatment of the derivation of the possible baryonic states and their multiplets can be put on a firmer quantitative footing with the help of SU(6) quark symmetry, we refer here to the literature (see, e.g., [Cl79]).

The proton and neutron wave functions may be schematically written as

$$|\mathrm{p}^{\uparrow}\rangle = |\mathrm{u}^{\uparrow}\mathrm{u}^{\uparrow}\mathrm{d}^{\downarrow}\rangle \qquad |\mathrm{n}^{\uparrow}\rangle = |\mathrm{u}^{\downarrow}\mathrm{d}^{\uparrow}\mathrm{d}^{\uparrow}\rangle\,.$$

We now want to construct the symmetrised wave function. For a proton with, e.g., the z spin component $m_J=+1/2$, we may write the spin wave function as a product of the the spin wave function of one quark and that of the remaining pair:

$$\chi_{\mathrm{p}}(J=\tfrac{1}{2}, m_J=\tfrac{1}{2}) = \sqrt{2/3}\,\chi_{\mathrm{uu}}(1,1)\chi_{\mathrm{d}}(\tfrac{1}{2},-\tfrac{1}{2}) - \sqrt{1/3}\,\chi_{\mathrm{uu}}(1,0)\chi_{\mathrm{d}}(\tfrac{1}{2},\tfrac{1}{2})\,. \tag{15.3}$$

Here we have chosen to single out the d-quark and coupled the u-quark pair. (If we initially single out one of the u-quarks we obtain the same result, but the notation becomes much more complicated.) The factors in this equation are the Clebsch-Gordan coefficients for the coupling of spin 1 and spin 1/2. Replacing $\chi(1,0)$ by the correct spin triplet wave function $(\uparrow\downarrow+\downarrow\uparrow)/\sqrt{2}$ then yields in our spin-flavour notation

$$|\mathrm{p}^{\uparrow}\rangle = \sqrt{2/3}\,|\mathrm{u}^{\uparrow}\mathrm{u}^{\uparrow}\mathrm{d}^{\downarrow}\rangle - \sqrt{1/6}\,|\mathrm{u}^{\uparrow}\mathrm{u}^{\downarrow}\mathrm{d}^{\uparrow}\rangle - \sqrt{1/6}\,|\mathrm{u}^{\downarrow}\mathrm{u}^{\uparrow}\mathrm{d}^{\uparrow}\rangle\,. \tag{15.4}$$

This expression is still only symmetric in terms of the exchange of the first and second quarks, and not for two arbitrary quarks as we need. It can, however, be straightforwardly totally symmetrised by swapping the first and third as well as the second and third quarks in each term of this last equation and adding these new terms. With the correct normalisation factor the totally symmetric proton wave function is then

$$\begin{aligned}|\mathrm{p}^{\uparrow}\rangle = \frac{1}{\sqrt{18}}\,\{\; & 2\,|\mathrm{u}^{\uparrow}\mathrm{u}^{\uparrow}\mathrm{d}^{\downarrow}\rangle + 2\,|\mathrm{u}^{\uparrow}\mathrm{d}^{\downarrow}\mathrm{u}^{\uparrow}\rangle + 2\,|\mathrm{d}^{\downarrow}\mathrm{u}^{\uparrow}\mathrm{u}^{\uparrow}\rangle - |\mathrm{u}^{\uparrow}\mathrm{u}^{\downarrow}\mathrm{d}^{\uparrow}\rangle \\ & -|\mathrm{u}^{\uparrow}\mathrm{d}^{\uparrow}\mathrm{u}^{\downarrow}\rangle - |\mathrm{d}^{\uparrow}\mathrm{u}^{\uparrow}\mathrm{u}^{\downarrow}\rangle - |\mathrm{u}^{\downarrow}\mathrm{u}^{\uparrow}\mathrm{d}^{\uparrow}\rangle - |\mathrm{u}^{\downarrow}\mathrm{d}^{\uparrow}\mathrm{u}^{\uparrow}\rangle - |\mathrm{d}^{\uparrow}\mathrm{u}^{\downarrow}\mathrm{u}^{\uparrow}\rangle\;\}\,.\end{aligned} \tag{15.5}$$

The neutron wave function is trivially found by exchanging the u- and d-quarks:

$$\begin{aligned}|\mathrm{n}^{\uparrow}\rangle = \frac{1}{\sqrt{18}}\,\{\; & 2\,|\mathrm{d}^{\uparrow}\mathrm{d}^{\uparrow}\mathrm{u}^{\downarrow}\rangle + 2\,|\mathrm{d}^{\uparrow}\mathrm{u}^{\downarrow}\mathrm{d}^{\uparrow}\rangle + 2\,|\mathrm{u}^{\downarrow}\mathrm{d}^{\uparrow}\mathrm{d}^{\uparrow}\rangle - |\mathrm{d}^{\uparrow}\mathrm{d}^{\downarrow}\mathrm{u}^{\uparrow}\rangle \\ & -|\mathrm{d}^{\uparrow}\mathrm{u}^{\uparrow}\mathrm{d}^{\downarrow}\rangle - |\mathrm{u}^{\uparrow}\mathrm{d}^{\uparrow}\mathrm{d}^{\downarrow}\rangle - |\mathrm{d}^{\downarrow}\mathrm{d}^{\uparrow}\mathrm{u}^{\uparrow}\rangle - |\mathrm{d}^{\downarrow}\mathrm{u}^{\uparrow}\mathrm{d}^{\uparrow}\rangle - |\mathrm{u}^{\uparrow}\mathrm{d}^{\downarrow}\mathrm{d}^{\uparrow}\rangle\;\}\,.\end{aligned} \tag{15.6}$$

The nucleons have isospin 1/2 and so form an isospin doublet. A further doublet may be produced by combining two s-quarks with a light quark. This is schematically given by

$$|\Xi^{0\uparrow}\rangle = |\mathrm{u}^{\downarrow}\mathrm{s}^{\uparrow}\mathrm{s}^{\uparrow}\rangle \qquad |\Xi^{-\uparrow}\rangle = |\mathrm{d}^{\downarrow}\mathrm{s}^{\uparrow}\mathrm{s}^{\uparrow}\rangle\,. \tag{15.7}$$

The remaining quark combinations are an isospin triplet and a singlet:

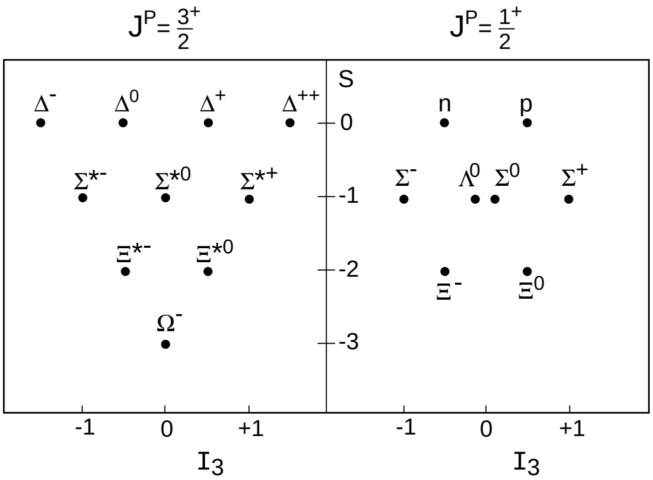


Fig. 15.4. The baryon $J^P = 3/2^+$ decuplet (*left*) and the $J^P = 1/2^+$ octet (*right*) in I_3 vs. S plots. In contradistinction to the mesonic case the baryon multiplets are solely composed of quarks. Antibaryons are purely composed of antiquarks and so form their own, equivalent antibaryon multiplets.

$$
\begin{aligned}
|\Sigma^{+\uparrow}\rangle &= |\mathrm{u}^{\uparrow}\mathrm{u}^{\uparrow}\mathrm{s}^{\downarrow}\rangle \\
|\Sigma^{0\uparrow}\rangle &= |\mathrm{u}^{\uparrow}\mathrm{d}^{\uparrow}\mathrm{s}^{\downarrow}\rangle \qquad |\Lambda^{0\uparrow}\rangle = |\mathrm{u}^{\uparrow}\mathrm{d}^{\downarrow}\mathrm{s}^{\uparrow}\rangle \\
|\Sigma^{-\uparrow}\rangle &= |\mathrm{d}^{\uparrow}\mathrm{d}^{\uparrow}\mathrm{s}^{\downarrow}\rangle\,.
\end{aligned}
\tag{15.8}
$$

Note that the uds quark combination appears twice here and depending upon the relative quark spins and isospins can correspond to two different particles. If the u and d spins and isospins couple to 1, as they do for the charged Σ baryons, then the above quark combination is a Σ^0. If they couple to zero we are dealing with a Λ^0. These two hyperons have a mass difference of about 80 MeV/c^2. This is evidence that a spin-spin interaction must also play an important role in the physics of the baryon spectrum. The eight $J^P = 1/2^+$ baryons are displayed in an I_3 vs. S plot in Fig. 15.4. Note again the threefold symmetry of the states.

15.3 Baryon Masses

The mass spectrum of the baryons is plotted in Fig. 15.5 against strangeness and isospin. The lowest energy levels are the $J^P = 1/2^+$ and $J^P = 3/2^+$ multiplets, as can be clearly seen. It is also evident that the baryon masses increase with the number of strange quarks, which we can put down to the larger mass of the s-quark. Furthermore we can see that the $J^P = 3/2^+$ baryons are about 300 MeV/c^2 heavier than their $J^P = 1/2^+$ equivalents. As

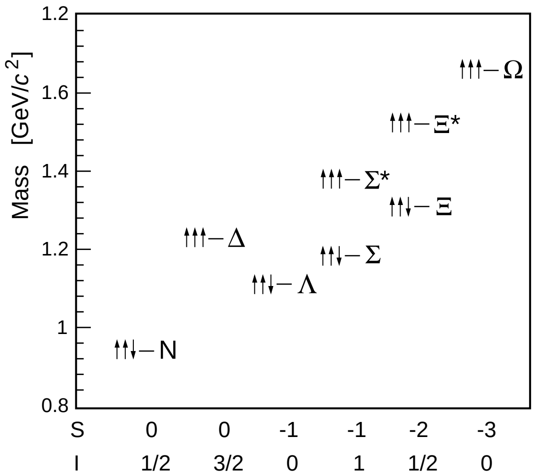


Fig. 15.5. The masses of the decuplet and octet baryons plotted against their strangeness S and isospin I. The angular momenta J of the various baryons are shown through arrows. The $J^P = 3/2^+$ decuplet baryons lie significantly above their $J^P = 1/2^+$ octet partners.

was the case with the mesons, this effect can be traced back to a spin-spin interaction

$$V_{\rm ss}(\mathrm{q}_i\mathrm{q}_j) = \frac{4\pi}{9}\frac{\hbar^3}{c}\alpha_{\rm s}\frac{\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j}{m_i m_j}\,\delta(\boldsymbol{x})\,, \tag{15.9}$$

which is only important at short distances. The observant reader may notice that the 4/9 factor is only half that which we found for the quark-antiquark potential in the mesons (13.10), this is a result of QCD considerations. Eq. (15.9), it should be noted, describes only the interaction of two quarks with each other and so to describe the baryon mass splitting we need to sum the spin-spin interactions over all quark pairs. The easiest cases are those like the nucleons, the Δ's and the Ω where the constituent masses of all three quarks are the same. Then we just have to calculate the expectation values for the sums over $\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$. Denoting the total baryon spin by $\boldsymbol{S}$ and using the identity $\boldsymbol{S}^2 = (\boldsymbol{s}_1+\boldsymbol{s}_2+\boldsymbol{s}_3)^2$ we find in a similar way to (13.11):

$$\sum_{\substack{i,j=1\\ i<j}}^{3}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j = \frac{4}{\hbar^2}\sum_{\substack{i,j=1\\ i<j}}^{3}\boldsymbol{s}_i\cdot\boldsymbol{s}_j = \begin{cases} -3 \text{ for } S=1/2\,,\\ +3 \text{ for } S=3/2\,.\end{cases} \tag{15.10}$$

The spin-spin energy (mass) splitting for these baryons is then just

Table 15.1. The masses of the lightest baryons both from experiment and as fitted from (15.12). The fits were to the average values of the various multiplets and are in good agreement with the measured masses. Also included in this table are the lifetimes and most important decay channels of these baryons [PD98]. The four charged Δ resonances are not individually listed.

	S	I	Baryon	Mass [MeV/c^2] theor.	Mass [MeV/c^2] exp.	τ [s]	Primary decay channels	Decay type
Octet ($J^P = 1/2^+$)	0	1/2	p	939	938.3	stable?	—	—
			n		939.6	886.7	$\mathrm{pe}^-\overline{\nu}_\mathrm{e}$ 100 %	weak
	−1	0	Λ	1114	1115.7	$2.63 \cdot 10^{-10}$	$\mathrm{p}\pi^-$ 64.1 % $\mathrm{n}\pi^0$ 35.7 %	weak weak
		1	Σ^+	1179	1189.4	$0.80 \cdot 10^{-10}$	$\mathrm{p}\pi^0$ 51.6 % $\mathrm{n}\pi^+$ 48.3 %	weak weak
			Σ^0		1192.6	$7.4 \cdot 10^{-20}$	$\Lambda\gamma$ ≈ 100 %	elmgn.
			Σ^-		1197.4	$1.48 \cdot 10^{-10}$	$\mathrm{n}\pi^-$ 99.8 %	weak
	−2	1/2	Ξ^0	1327	1315	$2.90 \cdot 10^{-10}$	$\Lambda\pi^0$ ≈ 100 %	weak
			Ξ^-		1321	$1.64 \cdot 10^{-10}$	$\Lambda\pi^-$ ≈ 100 %	weak
Decuplet ($J^P = 3/2^+$)	0	3/2	Δ	1239	1232	$0.55 \cdot 10^{-23}$	$\mathrm{N}\pi$ 99.4 %	strong
	−1	1	Σ^{*+}	1381	1383	$1.7 \cdot 10^{-23}$	$\Lambda\pi$ 88 % $\Sigma\pi$ 12 %	strong strong
			Σ^{*0}		1384			
			Σ^{*-}		1387			
	−2	1/2	Ξ^{*0}	1529	1532	$7 \cdot 10^{-23}$	$\Xi\pi$ ≈ 100 %	strong
			Ξ^{*-}		1535			
	−3	0	Ω^-	1682	1672.4	$0.82 \cdot 10^{-10}$	$\Lambda\mathrm{K}^-$ 68 % $\Xi^0\pi^-$ 23 % $\Xi^-\pi^0$ 9 %	weak weak weak

$$
\Delta M_{\mathrm{ss}} = \begin{cases} -3 \cdot \dfrac{4}{9}\dfrac{\hbar^3}{c^3}\dfrac{\pi\alpha_\mathrm{s}}{m_{\mathrm{u,d}}^2}\,|\psi(0)|^2 & \text{for the nucleons}\,, \\ +3 \cdot \dfrac{4}{9}\dfrac{\hbar^3}{c^3}\dfrac{\pi\alpha_\mathrm{s}}{m_{\mathrm{u,d}}^2}\,|\psi(0)|^2 & \text{for the } \Delta \text{ states}\,, \\ +3 \cdot \dfrac{4}{9}\dfrac{\hbar^3}{c^3}\dfrac{\pi\alpha_\mathrm{s}}{m_\mathrm{s}^2}\,|\psi(0)|^2 & \text{for the } \Omega \text{ baryon}\,. \end{cases} \tag{15.11}
$$

Here $|\psi(0)|^2$ is the probability that two quarks are at the same place. Somewhat more complicated expressions may be obtained for those baryons made up of a mixture of heavier s- and lighter u- or d-quarks (see the exercises).

With the help of this mass splitting formula a general expression for the masses of all the $\ell = 0$ baryons may be written:

$$M = \sum_i m_i + \Delta M_{\mathrm{ss}} \,. \tag{15.12}$$

The three unknowns here, i.e., $m_{\mathrm{u,d}}$, m_{s} and $\alpha_{\mathrm{s}}|\psi(0)|^2$, may be obtained by fitting to the experimental masses. As with the mesons we assume that $\alpha_{\mathrm{s}}|\psi(0)|^2$ is roughly the same for all of the baryons. We so obtain the following constituent quark masses: $m_{\mathrm{u,d}} \approx 363\,\mathrm{MeV}/c^2$, $m_{\mathrm{s}} \approx 538\,\mathrm{MeV}/c^2$ [Ga81]. The fitted baryon masses are within 1 % of their true values. (Table 15.1). The constituent quark masses obtained from such studies of baryons are a little larger than their mesonic counterparts. This is not necessarily a contradiction since constituent quark masses are generated by the dynamics of the quark-gluon interaction and the effective interactions of a three-quark system will not be identical to those of a quark-antiquark one.

15.4 Magnetic Moments

The constituent quark model is satisfyingly successful when its predictions for baryonic magnetic moments are compared with the results of experiment. In Dirac theory the magnetic moment μ of a point particle with mass M and spin 1/2 is

$$\mu_{\mathrm{Dirac}} = \frac{e\hbar}{2M} \,. \tag{15.13}$$

This relationship has been experimentally confirmed for both the electron and the muon. If the proton were an elementary particle without any substructure, then its magnetic moment should be one nuclear magneton:

$$\mu_{\mathrm{N}} = \frac{e\hbar}{2M_{\mathrm{p}}} \,. \tag{15.14}$$

Experimentally, however, the magnetic moment of the proton is measured to be $\mu_{\mathrm{p}} = 2.79\,\mu_{\mathrm{N}}$.

Magnetic moments in the quark model. The proton magnetic moment in the ground state, with $\ell = 0$, is a simple vectorial sum of the magnetic moments of the three quarks:

$$\boldsymbol{\mu}_{\mathrm{p}} = \boldsymbol{\mu}_{\mathrm{u}} + \boldsymbol{\mu}_{\mathrm{u}} + \boldsymbol{\mu}_{\mathrm{d}} \,. \tag{15.15}$$

The proton magnetic moment μ_{p} then has the expectation value

$$\mu_{\mathrm{p}} = \langle \boldsymbol{\mu}_{\mathrm{p}} \rangle = \langle \psi_{\mathrm{p}} | \boldsymbol{\mu}_{\mathrm{p}} | \psi_{\mathrm{p}} \rangle \,, \tag{15.16}$$

where ψ_p is the total antisymmetric quark wave function of the proton. To obtain μ_p we merely require the spin part of the wave function, χ_p. From (15.3) we thus deduce

$$\mu_p = \frac{2}{3}(\mu_u + \mu_u - \mu_d) + \frac{1}{3}\mu_d = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d \,, \tag{15.17}$$

where $\mu_{u,d}$ are the quark magnetons:

$$\mu_{u,d} = \frac{z_{u,d}\, e\hbar}{2m_{u,d}} \,. \tag{15.18}$$

The other $J^P = 1/2^+$ baryons with two identical quarks may be described by (15.17) with a suitable change of quark flavours. The neutron, for example, has a magnetic moment

$$\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u \tag{15.19}$$

and analogously for the Σ^+ we have

$$\mu_{\Sigma^+} = \frac{4}{3}\mu_u - \frac{1}{3}\mu_s \,. \tag{15.20}$$

The situation is a little different for the Λ^0. As we know this hyperon contains a u- and a d-quark whose spins are coupled to 0 and so contribute neither to the spin nor to the magnetic moment of the baryon (Sect. 15.2). Hence both the spin and the magnetic moment of the Λ^0 are determined solely by the s-quark:

$$\mu_\Lambda = \mu_s \,. \tag{15.21}$$

To the extent that the u and d constituent quark masses can be set equal to each other we have $\mu_u = -2\mu_d$ and may then write the proton and neutron magnetic moments as follows

$$\mu_p = \frac{3}{2}\mu_u \,, \qquad \mu_n = -\mu_u \,. \tag{15.22}$$

We thus obtain the following prediction for their ratio

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3} \,, \tag{15.23}$$

which is in excellent agreement with the experimental result of -0.685.

The absolute magnetic moments can only be calculated if we can specify the quark masses. Let us first, however, look at this problem the other way round and use the measured value of μ_p to determine the quark masses. From

$$\mu_p = 2.79\,\mu_N = 2.79\frac{e\hbar}{2M_p} \tag{15.24}$$

and

$$\mu_{\mathrm{p}} = \frac{3}{2}\mu_{\mathrm{u}} = \frac{e\hbar}{2m_{\mathrm{u}}} \tag{15.25}$$

we obtain

$$m_{\mathrm{u}} = \frac{M_{\mathrm{p}}}{2.79} = 336\ \mathrm{MeV}/c^2\ , \tag{15.26}$$

which is very close indeed to the mass we found in Sect. 15.3 from the study of the baryon spectrum.

Measuring the magnetic moments. The agreement between the experimental values of the hyperon magnetic moments with the predictions of the quark model is impressive (Table 15.2). Our ability to measure the magnetic moments of many of the short lived hyperons ($\tau \approx 10^{-10}$ s) is due to a combination of two circumstances: hyperons produced in nucleon-nucleon interactions are polarised and the weak interaction violates parity maximally. In consequence the angular distributions of their decay products are strongly dependent upon the direction of the hyperons' spins (i.e., their polarisations).

Let us clarify these remarks by studying how the magnetic moment of the Λ^0 is experimentally measured. Note that this is the most easily determined of the hyperon magnetic moments. The decay

$$\Lambda^0 \to \mathrm{p} + \pi^-$$

Table 15.2. Experimental and theoretical values of the baryon magnetic moments [La91, PD98]. The measured values of the p, n and Λ^0 moments are used to predict those of the other baryons. The Σ^0 hyperon has a very short lifetime ($7.4 \cdot 10^{-20}$ s) and decays electromagnetically via $\Sigma^0 \to \Lambda^0 + \gamma$. For this particle the transition matrix element $\langle \Lambda^0 | \mu | \Sigma^0 \rangle$ is given in place of its magnetic moment.

Baryon	μ/μ_{N} (Experiment)		Quark model:	μ/μ_{N}
p	+2.792 847 386	± 0.000 000 063	$(4\mu_{\mathrm{u}} - \mu_{\mathrm{d}})/3$	—
n	−1.913 042 75	± 0.000 000 45	$(4\mu_{\mathrm{d}} - \mu_{\mathrm{u}})/3$	—
Λ^0	−0.613	± 0.004	μ_{s}	—
Σ^+	+2.458	± 0.010	$(4\mu_{\mathrm{u}} - \mu_{\mathrm{s}})/3$	+2.67
Σ^0			$(2\mu_{\mathrm{u}} + 2\mu_{\mathrm{d}} - \mu_{\mathrm{s}})/3$	+0.79
$\Sigma^0 \to \Lambda^0$	−1.61	± 0.08	$(\mu_{\mathrm{d}} - \mu_{\mathrm{u}})/\sqrt{3}$	−1.63
Σ^-	−1.160	± 0.025	$(4\mu_{\mathrm{d}} - \mu_{\mathrm{s}})/3$	−1.09
Ξ^0	−1.250	± 0.014	$(4\mu_{\mathrm{s}} - \mu_{\mathrm{u}})/3$	−1.43
Ξ^-	−0.650 7	± 0.002 5	$(4\mu_{\mathrm{s}} - \mu_{\mathrm{d}})/3$	−0.49
Ω^-	−2.02	± 0.05	$3\mu_{\mathrm{s}}$	−1.84

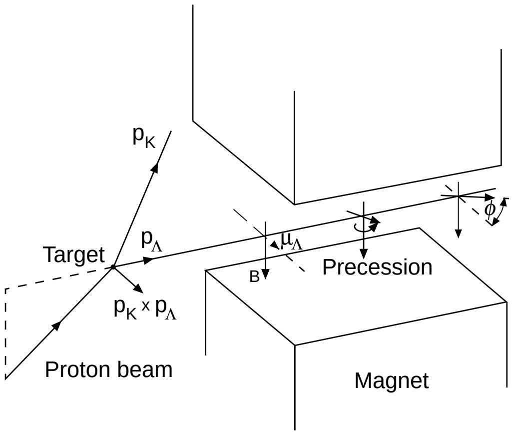


Fig. 15.6. Sketch of the measurement of the magnetic moment of the Λ^0. The hyperon is generated by the interaction of a proton coming in from the left with a proton in the target. The spin of the Λ^0 is, for reasons of parity conservation, perpendicular to the production plane. The Λ^0 then passes through a magnetic field which is orthogonal to the particle's spin. After traversing a distance d in the magnetic field the spin has precessed through an angle ϕ.

is rather simple to identify and has the largest branching ratio (64 %). If the Λ^0 spin is, say, in the positive $\hat{z}$ direction, then the proton will most likely be emitted in the negative $\hat{z}$ direction, in accord with the angular distribution

$$W(\theta) \propto 1 + \alpha \cos\theta \qquad \text{where} \quad \alpha \approx 0.64\,. \tag{15.27}$$

The angle θ is the angle between the spin of the Λ^0 and the momentum of the proton. The parameter α depends upon the strength of the interference of those terms with orbital angular momentum $\ell = 0$ and $\ell = 1$ in the p-π^- system and its size must be determined by experiment.

The asymmetry in the emitted protons then fixes the Λ^0 polarisation. Highly polarised Λ^0 particles may be obtained from the reaction

$$\mathrm{p} + \mathrm{p} \to \mathrm{K}^+ + \Lambda^0 + \mathrm{p}\,.$$

As shown in Fig. 15.6, the spin of the Λ^0 is perpendicular to the production plane defined by the path of the incoming proton and that of the Λ^0 itself. This is because only this polarisation direction conserves parity, which is conserved in the strong interaction.

If the Λ^0 baryon traverses a distance d in a magnetic field $\boldsymbol{B}$, where the field is perpendicular to the hyperon's spin, then its spin precesses with the Larmor frequency

$$\omega_\mathrm{L} = \frac{\boldsymbol{\mu}_\Lambda \boldsymbol{B}}{\hbar} \tag{15.28}$$

through the angle

$$\phi = \omega_{\rm L} \Delta t = \omega_{\rm L} \frac{d}{v} \,, \tag{15.29}$$

where v is the speed of the Λ^0 (this may be reconstructed by measuring the momenta of its decay products, i.e., a proton and a pion). The most accurate results may be obtained by reversing the magnetic field and measuring the angle $2 \cdot \phi$ which is given by the difference between the directions of the Λ^0 spins (after crossing the various magnetic fields). This trick neatly eliminates most of the systematic errors. The magnetic moment is thus found to be [PD94]

$$\mu_\Lambda = (-0.613 \pm 0.004)\, \mu_{\rm N} \,. \tag{15.30}$$

If we suppose that the s-constituent quark is a Dirac particle and that its magnetic moment obeys (15.18), then we see that this result for μ_Λ is consistent with a strange quark mass of 510 MeV/c^2.

The magnetic moments of many of the hyperons have been measured in a similar fashion to the above. There is an additional complication for the charged hyperons in that their deflection by the magnetic field must be taken into account if one wants to study spin precession effects. The best results have been obtained at Fermilab and are listed in Table 15.2. These results are compared with quark model predictions. The results for the proton, the neutron and the Λ^0 were used to fix all the unknown parameters and so predict the other magnetic moments. The results of the experiments agree with the model predictions to within a few percent.

These results support our constituent quark picture in two ways: firstly the constituent quark masses from our mass formula and those obtained from the above analysis of the magnetic moments agree well with each other and secondly the magnetic moments themselves are consistent with the quark model.

It should be noted, however, that the deviations of the experimental values from the predictions of the model show that the constituent quark magnetic moments alone do not suffice to describe the magnetic moments of the hyperons exactly. Further effects, such as relativistic ones and those due to the quark orbital angular momenta, must be taken into account.

15.5 Semileptonic Baryon Decays

The weak decays of the baryons all follow the same pattern. A quark emits a virtual $W^\pm$ boson and so changes its weak isospin and turns into a lighter quark. The $W^\pm$ decays into a lepton-antilepton pair or, if its energy suffices, a quark-antiquark pair. In the decays into a quark-antiquark pair we actually measure one or more mesons in the final state. These decays cannot be exactly calculated because of the strong interaction's complications. Matters are simpler for semileptonic decays. The rich data available to us from semileptonic baryon decays have made a decisive contribution to our current understanding of the weak interaction as formulated in the generalised Cabibbo theory.

We now want to attempt to describe the weak decays of the baryons using our knowledge of the weak interaction from Chap. 10. The weak decays take place essentially at the quark level, but free quarks do not exist and experiments always see hadrons. We must therefore try to interpret hadronic observables within the framework of the fundamental theory of the weak interaction. We will start by considering the β-decay of the neutron, since this has been thoroughly investigated in various experiments. It will then be only a minor matter to extend the formalism to the semileptonic decays of the hyperons and to nuclear β-decays.

We have seen from leptonic decays such as $\mu^- \to \mathrm{e}^- + \overline{\nu}_\mathrm{e} + \nu_\mu$ that the weak interaction violates parity conservation maximally, which must mean that the coupling constants for the vector and axial vector terms are of the same size. Since neutrinos are left handed and antineutrinos are right handed the coupling constants must have opposite signs (V−A theory). The weak decay of a hadron really means that a confined quark has decayed. It is therefore essential to take the quark wave function of the hadron into account. Furthermore strong interaction effects of virtual particles cannot be neglected: although the effective electromagnetic coupling constant is for reasons of charge conservation not altered by the cloud of sea quarks and gluons, the weak coupling is indeed so changed. In what follows we will initially take the internal structure of the hadrons into account and then discuss the coupling constants.

β-decay of the neutron. The β-decay of a free neutron

$$\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e} \tag{15.31}$$

(maximum electron energy $E_0 = 782\,\mathrm{keV}$, lifetime 15 minutes) is a rich source of precise data about the low energy behaviour of the weak interaction.

To find the form of the β-spectrum and the coupling constants of neutron β-decay we consider the decay probability. This may be calculated from the golden rule in the usual fashion. If the electron has energy E_e, then the decay rate is

$$\mathrm{d}W(E_\mathrm{e}) = \frac{2\pi}{\hbar}\,|\mathcal{M}_{fi}|^2\,\frac{\mathrm{d}\varrho_f(E_0, E_\mathrm{e})}{\mathrm{d}E_\mathrm{e}}\mathrm{d}E_\mathrm{e}\;, \tag{15.32}$$

where $\mathrm{d}\varrho_f(E_0, E_\mathrm{e})/\mathrm{d}E_\mathrm{e}$ is the density of antineutrino-electron final states with total energy E_0 and the electron having energy E_e and $\mathcal{M}_{fi}$ is the matrix element for the β-decay.

Vector transitions. A β-decay which takes place through a vector coupling is called a *Fermi transition.* The direction of the quark's spin is unaltered in these decays. The change of a d- into a u-quark is described by the ladder operator of weak isospin T_+ which changes a state with $T=-1/2$ into one with $T=+1/2$.

The matrix element for neutron β-decay has a leptonic and a quark part. Conservation of angular momentum prevents any interference between vector and axial vector transitions, i.e., a quark vector transition necessarily implies a leptonic vector transition. Since we already have $c_{\rm V} = -c_{\rm A} = 1$ for leptons, we do not need to worry further about their part of the matrix element.

The matrix element for Fermi decays may then be written as

$$|\mathcal{M}_{fi}|_{\rm F} \;=\; \frac{G_{\rm F}}{V}\, c_{\rm V}\; |\langle\, {\rm uud}\,|\sum_{i=1}^{3} T_{i,+}|\,{\rm udd}\,\rangle| \tag{15.33}$$

where the sum is over the three quarks. According to the definition (10.4) the Fermi constant $G_{\rm F}$ includes the propagator term and the coupling to the leptons. The initial neutron state has the wave function $|{\rm udd}\,\rangle$ and the final state is described by the quark combination $|{\rm uud}\,\rangle$. The wave functions of the electron and the antineutrino can each be replaced by $1/\sqrt{V}$, since we have $pR/\hbar \ll 1$.

The u- and d-quarks in the proton and neutron wave functions are eigenstates of strong isospin. In β-decay we need to consider the eigenstates of the weak interaction. We therefore recall that while the ladder operators $I_\pm$ of the strong force map $|{\rm u}\rangle$ and $|{\rm d}\rangle$ onto each other, the $T_\pm$ operators connect the $|{\rm u}\rangle$ and $|{\rm d}'\rangle$ quark states. The overlap between $|{\rm d}\rangle$ and $|{\rm d}'\rangle$ is, according to (10.16), fixed by the cosine of the Cabibbo angle. Hence

$$\langle {\rm u}|T_+|{\rm d}\rangle = \langle {\rm u}|I_+|{\rm d}\rangle \cdot \cos\theta_{\rm C} \qquad \text{where} \;\; \cos\theta_{\rm C} \approx 0.98\,. \tag{15.34}$$

The vector component of the matrix element is then

$$\mathcal{M}_{fi} = \frac{G_{\rm F}}{V}\cos\theta_{\rm C}\cdot c_{\rm V}\,\langle\,{\rm uud}\,|\sum_{i=1}^{3} I_{i,+}\,|{\rm udd}\,\rangle = \frac{G_{\rm F}}{V}\cos\theta_{\rm C}\cdot c_{\rm V}\cdot 1\,. \tag{15.35}$$

Here we have employed the fact that the sum $\langle{\rm uud}|\sum_i I_{i,+}|{\rm udd}\rangle$ must be unity since the operator $\sum_i I_{i,+}$ applied to the quark wave function of the neutron just gives the quark wave function of the proton. This follows from isospin conservation in the strong interaction and may be straightforwardly verified with the help of (15.5) and (15.6). We thus see that the Fermi matrix element is independent of the internal structure of the nucleon.

Axial transitions. Those β-decays that take place as a result of an axial vector coupling are called *Gamow-Teller transitions*. In such cases the direction of the fermion spin flips over. The matrix element depends upon the overlap of the spin densities of the particles carrying the weak charge in the initial and final states. The transition operator is then $c_{\rm A}\, T_+\boldsymbol{\sigma}$.

The universality of the weak interaction means that this result should also hold for free point quarks. Since quarks are always trapped inside hadrons, we need to consider the internal structure of the nucleon if we want to calculate such matrix elements. From the constituent quark model we have

$$|\mathcal{M}_{fi}|_{\mathrm{GT}} = \frac{G_{\mathrm{F}}}{V} c_{\mathrm{A}} \, |\langle \, \mathrm{uud} \, | \sum_{i=1}^{3} T_{i,+} \boldsymbol{\sigma} | \, \mathrm{udd} \, \rangle| \, . \tag{15.36}$$

Since the squares of the expectation values of the components of σ are equal to each other, $\langle \sum_i \sigma_{i,x} \rangle^2 = \langle \sum_i \sigma_{i,y} \rangle^2 = \langle \sum_i \sigma_{i,z} \rangle^2$, it is sufficient to calculate the expectation value of $\sigma_z = \langle \, \mathrm{uud} \, | \sum_i I_{i,+} \sigma_{i,z} | \, \mathrm{udd} \, \rangle$. One finds from (15.5), (15.6) and some tedious arithmetic that

$$\langle \, \mathrm{uud} \, | \sum_i I_{i,+} \sigma_{i,z} | \, \mathrm{udd} \, \rangle = \frac{5}{3} \, . \tag{15.37}$$

The total matrix element. In experiments we measure the properties of the nucleon, such as its spin, and not those of the quarks. To compare theory with experiment we must therefore reformulate the matrix element so that all operators act upon the *nucleon* wave function. The square of the neutron decay matrix element may be written as

$$|\mathcal{M}_{fi}|^2 = \frac{g_{\mathrm{V}}^2}{V^2} \, |\langle \, \mathrm{p} \, | I_+ | \, \mathrm{n} \, \rangle|^2 + \frac{g_{\mathrm{A}}^2}{V^2} \, |\langle \, \mathrm{p} \, | I_+ \boldsymbol{\sigma} | \, \mathrm{n} \, \rangle|^2 \, . \tag{15.38}$$

We stress that I_+ and $\boldsymbol{\sigma}$ now act upon the wave function of the nucleon. The quantities g_{V} and g_{A} are those which are measured in neutron β-decay and describe the absolute strengths of the vector and axial vector contributions. They contain the product of the weak charges at the leptonic and hadronic vertices.

Since the proton and the neutron form an isospin doublet, (15.38) may be written as

$$|\mathcal{M}_{fi}|^2 = (g_{\mathrm{V}}^2 + 3 g_{\mathrm{A}}^2)/V^2 \, . \tag{15.39}$$

We note that the factor of 3 in the axial vector part is due to the expectation value of the spin operator $\boldsymbol{\sigma}^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$.

In the constituent quark model g_{V} and g_{A} are related to the quark dependent coupling constants c_{V} and c_{A} as follows:

$$g_{\mathrm{V}} = G_{\mathrm{F}} \, \cos\theta_{\mathrm{C}} \, c_{\mathrm{V}} \, , \tag{15.40}$$

$$g_{\mathrm{A}} \approx G_{\mathrm{F}} \, \cos\theta_{\mathrm{C}} \, \frac{5}{3} \, c_{\mathrm{A}} \, . \tag{15.41}$$

The Fermi matrix element (15.35) is independent of the internal structure of the neutron and (15.40) is as exact as the isospin symmetry of the proton and the neutron. The axial vector coupling, on the other hand, does depend upon the structure of the nucleon. In the constituent quark model it is given by (15.41). It is important to understand that the factor of 5/3 is merely an estimate, since the constituent quark model only gives us an approximation of the nucleon wave function.

The neutron lifetime. The lifetime is given by the inverse of the total decay probability per unit time:

$$\frac{1}{\tau} = \int_{m_e c^2}^{E_0} \frac{\mathrm{d}W}{\mathrm{d}E_\mathrm{e}} \mathrm{d}E_\mathrm{e} = \int_{m_e c^2}^{E_0} \frac{2\pi}{\hbar} |\mathcal{M}_{fi}|^2 \frac{\mathrm{d}\varrho_f(E_0, E_\mathrm{e})}{\mathrm{d}E_\mathrm{e}} \mathrm{d}E_\mathrm{e} \,. \tag{15.42}$$

Assuming that the matrix element is independent of the energy, we can pull it outside the integral. The state density $\varrho_f(E_0, E_\mathrm{e})$ may, in analogy to (4.18) and (5.21), be written as

$$\mathrm{d}\varrho_f(E_0, E_\mathrm{e}) = \frac{(4\pi)^2}{(2\pi\hbar)^6} \, p_\mathrm{e}^2 \, \frac{\mathrm{d}p_\mathrm{e}}{\mathrm{d}E_\mathrm{e}} \, p_\nu^2 \, \frac{\mathrm{d}p_\nu}{\mathrm{d}E_0} \, V^2 \, \mathrm{d}E_\mathrm{e} \,, \tag{15.43}$$

where we have taken into account that we here have an electron and a neutrino and hence a 2-particle state density and V is the volume in which the wave functions of the electron and of the neutrino are normalised. Since this normalisation enters the matrix element (15.39) via a $1/V^2$ factor, the decay probability is independent of V.

In (15.42) we only integrate over the electron spectrum and so we need the density of states for a total energy E_0 with a fixed electron energy E_e. Neglecting recoil effects we have $E_0 = E_\mathrm{e} + E_\nu$ and hence $\mathrm{d}E_0 = \mathrm{d}E_\nu$. Using the relativistic energy-momentum relation $E^2 = p^2c^2 + m^2c^4$ we thus find

$$p_\mathrm{e}^2 \mathrm{d}p_\mathrm{e} \;=\; \frac{1}{c^2} p_\mathrm{e} E_\mathrm{e} \, \mathrm{d}E_\mathrm{e} = \frac{1}{c^3} E_\mathrm{e} \sqrt{E_\mathrm{e}^2 - m_\mathrm{e}^2 c^4} \, \mathrm{d}E_\mathrm{e} \tag{15.44}$$

and an analogous relation for the neutrino. Assuming that the neutrino is massless we obtain

$$\mathrm{d}\varrho_f(E_0, E_\mathrm{e}) = (4\pi)^2 \, V^2 \, \frac{E_\mathrm{e}\sqrt{E_\mathrm{e}^2 - m_\mathrm{e}^2 c^4} \cdot (E_0 - E_\mathrm{e})^2}{(2\pi\hbar c)^6} \, \mathrm{d}E_\mathrm{e} \,. \tag{15.45}$$

To find the lifetime τ we now need to carry out the integral (15.42). It is usual to normalise the energies in terms of the electron rest mass and so define

$$f(E_0) \;=\; \int_1^{\mathcal{E}_0} \mathcal{E}_\mathrm{e} \sqrt{\mathcal{E}_\mathrm{e}^2 - 1} \cdot (\mathcal{E}_0 - \mathcal{E}_\mathrm{e})^2 \, \mathrm{d}\mathcal{E}_\mathrm{e} \qquad \text{where} \quad \mathcal{E} = E/m_\mathrm{e}c^2. \tag{15.46}$$

Together with (15.39) this leads to

$$\frac{1}{\tau} = \frac{m_\mathrm{e}^5 c^4}{2\pi^3 \hbar^7} \cdot (g_\mathrm{V}^2 + 3g_\mathrm{A}^2) \cdot f(E_0) \,. \tag{15.47}$$

For ($E_0 \gg m_\mathrm{e}c^2$) we have

$$f(E_0) \approx \frac{\mathcal{E}_0^5}{30} \tag{15.48}$$

and so

$$\frac{1}{\tau} \approx \frac{1}{\hbar^7 c^6} \cdot (g_{\mathrm{V}}^2 + 3g_{\mathrm{A}}^2) \cdot \frac{E_0^5}{60\pi^3} . \tag{15.49}$$

This decrease of the lifetime as the fifth power of E_0 is called *Sargent's rule.*

In neutron decays E_0 is roughly comparable to $m_{\mathrm{e}}c^2$ and the approximation (15.48) is not applicable. The decay probability is roughly half the size of (15.49):

$$\frac{1}{\tau_{\mathrm{n}}} \approx \frac{1}{\hbar^7 c^6} \cdot (g_{\mathrm{V}}^2 + 3g_{\mathrm{A}}^2) \cdot \frac{E_0^5}{60\pi^3} \cdot 0.47 \, . \tag{15.50}$$

Experimental results. The neutron lifetime has been measured very precisely in recent years. The storage of ultra cold neutrons has been a valuable tool in these experiments [Ma89, Go94a]. Extremely slow neutrons can be stored between solid walls which represent a potential barrier. The neutrons are totally reflected since the refraction index in solid matter is smaller than that in air [Go79]. With such storage cells the lifetime of the neutron may be determined by measuring the number of neutrons in the cell as a function of time. To do this one opens the storage cell for a specific time to a cold neutron beam of a known, constant intensity. The cell is then closed and left undisturbed until after a certain time it is opened again and the remaining neutrons are counted with a neutron detector. The experiment is repeated for various storage times. The exponential decay in the number of neutrons in the cell (together with knowledge of the leakage rate from the cell) gives us the neutron lifetime. The average of the most recent measurements of the neutron lifetime is [PD98]

$$\tau_{\mathrm{n}} = 886.7 \pm 1.9 \,\mathrm{s} \, . \tag{15.51}$$

To individually determine g_{A} and g_{V} we need to measure a second quantity. The decay asymmetry of polarised neutrons is a good candidate here. This comes from the parity violating properties of the weak interaction: the axial vector part emits electrons anisotropically while the vector contribution is spherically symmetric.[2] The number of electrons that are emitted in the direction of the neutron spin $N^{\uparrow\uparrow}$ is smaller than the number $N^{\uparrow\downarrow}$ emitted in the opposite direction. The asymmetry A is defined by

$$\frac{N^{\uparrow\uparrow} - N^{\uparrow\downarrow}}{N^{\uparrow\uparrow} + N^{\uparrow\downarrow}} = \beta \cdot A \qquad \text{where} \quad \beta = \frac{v}{c} \, . \tag{15.52}$$

This asymmetry is connected to

$$\lambda = \frac{g_{\mathrm{A}}}{g_{\mathrm{V}}} \tag{15.53}$$

[2] The discovery of parity violation in the weak interaction was through the anisotropic emission of electrons in the β-decay of atomic nuclei [Wu57].

by

$$A = -2\frac{\lambda(\lambda+1)}{1+3\lambda^2}\,. \tag{15.54}$$

The asymmetry experiments are also best performed with ultra low energy neutrons. An electron spectrometer with an extremely high spatial resolution is needed. Such measurements yield [PD98]

$$A = -0.1162 \;\pm\; 0.0013\,. \tag{15.55}$$

Combining this information we have

$$\begin{aligned} \lambda &= -1.267 \pm 0.004\,, \\ g_{\rm V}/(\hbar c)^3 &= +1.153 \cdot 10^{-5}\,\mathrm{GeV}^{-2}\,, \\ g_{\rm A}/(\hbar c)^3 &= -1.454 \cdot 10^{-5}\,\mathrm{GeV}^{-2}\,. \end{aligned} \tag{15.56}$$

A comparison with (15.40) yields very exactly $c_{\rm V} = 1$, which is the value we would expect for a point-like quark or lepton. The vector part of the interaction is conserved in weak baryon decays. This is known as *conservation of vector current* (CVC) and it is believed that this conservation is exact. It is considered to be as important as the conservation of electric charge in electromagnetism.

The axial vector term is on the other hand not that of a point-like Dirac particle. Rather than $\lambda = -5/3$ experiment yields $\lambda \approx -5/4$. The strong force alters the spin dependent part of the weak decay and the axial vector current is only partially conserved (PCAC = *partially conserved axial vector current*).

Semileptonic hyperon decays. The semileptonic decays of the hyperons can be calculated in a similar way to that of the neutron. Since the decay energies E_0 are typically two orders of magnitude larger than in the neutron decay, Sargent's rule (15.49) predicts that the hyperon lifetimes should be at least a factor of 10^{10} shorter. At the quark level these decays are all due to the decay $\mathrm{s} \to \mathrm{u} + \mathrm{e}^- + \overline{\nu}_{\mathrm{e}}$.

The two independent measurements to determine the semileptonic decay probabilities of the hyperons are their lifetimes τ and the branching ratio $V_{\rm semil.}$ of the semileptonic channels. From

$$\frac{1}{\tau} \propto |\mathcal{M}_{fi}|^2 \qquad \text{and} \quad V_{\rm semil.} \equiv \frac{|\mathcal{M}_{fi}|^2_{\rm semil.}}{|\mathcal{M}_{fi}|^2}$$

we have the relationship

$$\frac{V_{\rm semil.}}{\tau} \propto |\mathcal{M}_{fi}|^2_{\rm semil.}\,. \tag{15.57}$$

The lifetime may most easily be measured in production experiments. High energy proton or hyperon (e.g., Σ^-) beams with an energy of a few hundred GeV are fired at a fixed target and one detects the hyperons which are produced. One then calculates the average decay length of the secondary hyperons, i.e., the average distance between where they are produced (the target) and where they decay. This is done by measuring the tracks of the decay products with detectors which have a good spatial resolution and reconstructing the position where the hyperon decayed. The number of hyperons decreases exponentially with time and this is reflected in an exponential decrease in the number N of decay positions a distance l away from the target:

$$N = N_0\,\mathrm{e}^{-t/\tau} = N_0\,\mathrm{e}^{-l/L}\,. \tag{15.58}$$

The method of invariant masses must, of course, be used to identify which sort of hyperon has decayed. The average decay length L is then related to the lifetime τ as follows

$$L = \gamma v \tau\,, \tag{15.59}$$

where v is the velocity of the hyperon. With high beam energies the secondary hyperons can have time dilation factors $\gamma = E/mc^2$ of the order of 100. Since the hyperons typically have a lifetime of around 10^{-10} s the decay length will typically be a few metres – which may be measured to a good accuracy.

The measurement of the branching ratios is much more complicated. This is because the vast majority of decays are into hadrons (which may therefore be used to measure the decay length). The semileptonic decays are only about one thousandth of the total. This means that those few leptons must be detected with a very high efficiency and that background effects must be rigorously analysed.

The experiments are in fact sufficiently precise to put the Cabibbo theory to the test. The method is similar to that which we used in the case of the β-decay of the neutron. Using the relevant matrix element and phase space factors one calculates the decay probability of the decay under consideration. The calculation, which still contains c_{V} und c_{A}, is then compared with experiment.

Consider the strangeness-changing decay $\Xi^- \to \Lambda^0 + \mathrm{e}^- + \overline{\nu}_{\mathrm{e}}$. The matrix element for the Fermi decay is

$$|\mathcal{M}_{fi}|_{\mathrm{F}} \;=\; \frac{G_{\mathrm{F}}}{V}\;|\langle\,\mathrm{uds}\,|\sum_{i=1}^{3} T_{i,+}|\,\mathrm{dss}\,\rangle|\,, \tag{15.60}$$

where we have assumed that the coupling constant $c_{\mathrm{V}} = 1$ is unchanged. Applying the operator T_+ to the flavour eigenstate $|\mathrm{s}\rangle$ yields a linear combination of $|\mathrm{u}\rangle$ and $|\mathrm{c}\rangle$. Just as was the case for the β-decay of the neutron the matrix element thus contains a Cabibbo factor, here $\sin\theta_{\mathrm{C}}$. The Gamow-Teller matrix element is obtained from

$$|\mathcal{M}_{fi}|_{\mathrm{GT}} \;=\; \frac{g_{\mathrm{A}}}{g_{\mathrm{V}}}\frac{G_{\mathrm{F}}}{V}\;|\langle\,\mathrm{uds}\,|\sum_{i=1}^{3} T_{i,+}\boldsymbol{\sigma}_i|\,\mathrm{dss}\,\rangle|\,. \tag{15.61}$$

Of course the evaluation of the $\boldsymbol{\sigma}$ operator depends upon the wave functions of the baryons involved in the decay.

The analysis of the data confirms the assumption that the ratio $\lambda = g_{\mathrm{A}}/g_{\mathrm{V}}$ has the same value in both hyperon and neutron decays. The axial current is hence modified in the same way for all three light quark flavours.

15.6 How Good is the Constituent Quark Concept?

We introduced the concept of constituent quarks so as to describe the baryon mass spectrum as simply as possible. We thus viewed constituent quarks as the building blocks from which the hadrons can be constructed. This means, however, that we should be able to derive all the hadronic quantum numbers from these effective constituents. Furthermore we have silently assumed that we are entitled to treat constituent quarks as elementary particles, whose magnetic moments, just like the electrons', obey a Dirac relation (15.13). That these ideas work has been seen in the chapters treating the meson and baryon masses and the magnetic moments. Various approaches led us to constituent quark masses which were in good agreement with each other and furthermore the magnetic moments of the model were generally in very good agreement with experiment.

Constituent quarks are not, however, fundamental, elementary particles as we understand the term. This role is reserved for the "naked" valence quarks which are surrounded by a cloud of virtual gluons and quark-antiquark pairs. It is not at all obvious why constituent quarks may be treated as though they were elementary. Indeed we have seen the limitations of this approach: in all those phenomena where spin plays a part the structure of the constituent quark makes itself to some extent visible, for example in the magnetic moments of the hyperons with 2 or 3 s-quarks and also in the non-conserved axial vector current of the weak interaction. The picture of hadrons as being composed of (Dirac particle) constituent quarks is just not up to describing such matters or indeed any process with high momentum transfer.

Problems

1. **Particle production and identification**
A liquid hydrogen target is bombarded with a $|\boldsymbol{p}|$ =12 GeV/c proton beam. The momenta of the reaction products are measured in wire chambers inside a magnetic field. In one event six charged particle tracks are seen. Two of them go back to the interaction vertex. They belong to positively charged particles. The other tracks come from two pairs of oppositely charged particles. Each of these pairs appears "out of thin air" a few centimetres away from the interaction point. Evidently two electrically neutral, and hence unobservable, particles were created which later both decayed into a pair of charged particles.
a) Make a rough sketch of the reaction (the tracks).
b) Use Tables 14.2, 14.3 and 15.1 as well as [PD94] to discuss which mesons and baryons have lifetimes such that they could be responsible for the two observed decays. How many decay channels into two charged particles are there?
c) The measured momenta of the decay pairs were:
1) $|\boldsymbol{p}_+| = 0.68\,\text{GeV}/c$, $|\boldsymbol{p}_-| = 0.27\,\text{GeV}/c$, $\sphericalangle(\boldsymbol{p}_+, \boldsymbol{p}_-) = 11°$;
2) $|\boldsymbol{p}_+| = 0.25\,\text{GeV}/c$, $|\boldsymbol{p}_-| = 2.16\,\text{GeV}/c$, $\sphericalangle(\boldsymbol{p}_+, \boldsymbol{p}_-) = 16°$.
The relative errors of these measurements are about 5 %. Use the method of invariant masses (15.1) to see which of your hypothesis from b) are compatible with these numbers.
d) Using these results and considering all applicable conservation laws produce a scheme for all the particles produced in the reaction. Is there a unique solution?

2. **Baryon masses**
Calculate expressions analogous to (15.11) for the mass shifts of the Σ and Σ^* baryons due to the spin-spin interaction. What value do you obtain for $\alpha_s|\psi(0)|^2$ if you use the constituent quark masses from Sec. 15.3?

3. **Isospin coupling**
The Λ hyperon decays almost solely into $\Lambda^0 \to \text{p} + \pi^-$ and $\Lambda^0 \to \text{n} + \pi^0$. Apply the rules for coupling angular momenta to isospin to estimate the ratio of the two decay probabilities.

4. **Muon capture in nuclei**
Negative muons are slowed down in a carbon target and then trapped in atomic 1s states. Their lifetime is then 2.02 μs which is less than that of the free muon (2.097 μs). Show that the difference in the lifetimes is due to the capture reaction $^{12}\text{C}+\mu^- \to {}^{12}\text{B} + \nu_\mu$. The mass difference between the ^{12}B and ^{12}C atoms is 13.37 MeV/c^2 and the lifetime of ^{12}B is 20.2 ms. ^{12}B has, in the ground state, the quantum numbers $J^P = 1^+$ and $\tau = 20.2$ ms. The rest mass of the electron and the nuclear charge may be neglected in the calculation of the matrix element.

5. **Quark mixing**
The branching ratios for the semileptonic decays $\Sigma^- \to \text{n} + \text{e}^- + \overline{\nu}_\text{e}$ and $\Sigma^- \to \Lambda^0 + \text{e}^- + \overline{\nu}_\text{e}$ are $1.02 \cdot 10^{-3}$ and $5.7 \cdot 10^{-5}$ respectively – a difference of more than an order of magnitude. Why is this? The decay $\Sigma^+ \to \text{n} + \text{e}^+ + \nu_\text{e}$ has not yet been observed (upper bound: $5 \cdot 10^{-6}$). How would you explain this?

6. **Parity**

a) The intrinsic parity of a baryon cannot be determined in an experiment; it is only possible to compare the parity of one baryon with that of another. Why is this?

b) It is conventional to ascribe a positive parity to the nucleon. What does this say about the deuteron's parity (see Sec. 16.2) and the intrinsic parities of the u- and d-quarks?

c) If one bombards liquid deuterium with negative pions, the latter are slowed down and may be captured into atomic orbits. How can one show that they cascade down into the 1s shell (K shell)?

d) A pionic deuterium atom in the ground state decays through the strong interaction via $\mathrm{d} + \pi^- \rightarrow \mathrm{n} + \mathrm{n}$. In which ${}^{2S+1}\mathrm{L}_J$ state may the two neutron system be? Note that the two neutrons are identical fermions and that angular momentum is conserved.

e) What parity from this for the pion? What parity would one expect from the quark model (see Chap. 14)?

f) Would it be inconsistent to assign a positive parity to the proton and a negative one to the neutron? What would then be the parities of the quarks and of the pion? Which convention is preferable? What are the parities of the Λ and the Λ_c according to the quark model?