

# Contents

<b>1. Introduction to scheduling</b> .....	5
1.1 Definition .....	5
1.2 Some areas of application .....	6
1.2.1 Problems related to production .....	6
1.2.2 Other problems .....	7
1.3 Shop environments .....	7
1.3.1 Scheduling problems without assignment .....	8
1.3.2 Scheduling and assignment problems with stages .....	8
1.3.3 General scheduling and assignment problems .....	9
1.4 Constraints .....	9
1.5 Optimality Criteria .....	12
1.5.1 Minimisation of a maximum function: "minimax" criteria .....	13
1.5.2 Minimisation of a sum function: "minisum" criteria .....	13
1.6 Typologies and notation of problems .....	14
1.6.1 Typologies of problems .....	14
1.6.2 Notation of problems .....	16
1.7 Project scheduling problems .....	17
1.8 Some fundamental notions .....	18
1.9 Basic scheduling algorithms .....	21
1.9.1 Scheduling rules .....	21
1.9.2 Some classical scheduling algorithms .....	22
<b>2. Complexity of problems and algorithms</b> .....	29
2.1 Complexity of problems .....	29
2.2 Complexity of algorithms .....	35
2.3 Application to scheduling .....	38
<b>3. Multicriteria Optimisation Theory</b> .....	43
3.1 MCDA and MCDM: the context .....	43
3.1.1 MultiCriteria Decision Making .....	44
3.1.2 MultiCriteria Decision Aid .....	44
3.2 Presentation of multicriteria optimisation theory .....	45
3.3 Definition of optimality .....	47

3.4	Geometric interpretation using dominance cones . . . . .	50
3.5	Classes of resolution methods . . . . .	52
3.6	Determination of Pareto optima . . . . .	54
3.6.1	Determination by convex combination of criteria . . . . .	54
3.6.2	Determination by parametric analysis . . . . .	60
3.6.3	Determination by means of the $\epsilon$ -constraint approach . . . . .	62
3.6.4	Use of the Tchebycheff metric . . . . .	66
3.6.5	Use of the weighted Tchebycheff metric . . . . .	69
3.6.6	Use of the augmented weighted Tchebycheff metric . . . . .	71
3.6.7	Determination by the goal-attainment approach . . . . .	76
3.6.8	Other methods for determining Pareto optima . . . . .	81
3.7	Multicriteria Linear Programming (MLP) . . . . .	82
3.7.1	Initial results . . . . .	83
3.7.2	Application of the previous results . . . . .	83
3.8	Multicriteria Mixed Integer Programming (MMIP) . . . . .	84
3.8.1	Initial results . . . . .	84
3.8.2	Application of the previous results . . . . .	85
3.8.3	Some classical algorithms . . . . .	87
3.9	The complexity of multicriteria problems . . . . .	90
3.9.1	Complexity results related to the solutions . . . . .	90
3.9.2	Complexity results related to objective functions . . . . .	91
3.9.3	Summary . . . . .	96
3.10	Interactive methods . . . . .	97
3.11	Goal programming . . . . .	98
3.11.1	Archimedian goal programming . . . . .	100
3.11.2	Lexicographical goal programming . . . . .	101
3.11.3	Interactive goal programming . . . . .	101
3.11.4	Reference goal programming . . . . .	102
3.11.5	Multicriteria goal programming . . . . .	102
<b>4.</b>	<b>An approach to multicriteria scheduling problems . . . . .</b>	<b>103</b>
4.1	Justification of the study . . . . .	103
4.1.1	Motivations . . . . .	103
4.1.2	Some examples . . . . .	104
4.2	Presentation of the approach . . . . .	107
4.2.1	Definitions . . . . .	107
4.2.2	Notation of multicriteria scheduling problems . . . . .	109
4.3	Classes of resolution methods . . . . .	110
4.4	Application of the process - an example . . . . .	112
4.5	Some complexity results for multicriteria scheduling problems . . . . .	113

<b>5. Single machine Just-in-Time scheduling problems</b> .....	119
5.1 Presentation of Just-in-Time (JiT) scheduling problems .....	119
5.2 Typology of JiT scheduling problems .....	120
5.2.1 Definition of the due dates .....	120
5.2.2 Definition of the JiT criteria .....	121
5.3 A new approach for JiT scheduling .....	123
5.4 Polynomially solvable problems .....	125
5.5 $\mathcal{NP}$ -hard problems .....	134
5.6 Open problems .....	144
<b>6. Single machine problems</b> .....	147
6.1 Polynomially solvable problems .....	147
6.1.1 Minimisation of $K$ increasing functions of the completion times .....	147
6.1.2 Minimisation of the average completion time .....	147
6.1.3 Minimisation of the average weighted completion time .....	150
6.1.4 Minimisation of crashing time costs .....	151
6.1.5 Minimisation of tool changing costs .....	154
6.1.6 Minimisation of due date based criteria .....	155
6.2 $\mathcal{NP}$ -hard problems .....	156
6.2.1 Minimisation of the average completion time .....	156
6.2.2 Minimisation of the average weighted completion time .....	157
6.2.3 Minimisation of crashing time costs .....	159
6.2.4 Minimisation of tool changing costs .....	162
6.3 Open problems .....	162
6.3.1 Minimisation of the average completion time .....	162
6.3.2 Minimisation of crashing time costs .....	162
6.3.3 Minimisation of due dates based criteria .....	163
<b>7. Shop problems</b> .....	165
7.1 Two-machine flowshop problems .....	165
7.1.1 The $F2 prmu Lex(C_{max}, \bar{C})$ problem .....	165
7.1.2 The $F2 prmu F_{\ell}(C_{max}, \bar{C})$ problem .....	178
7.1.3 The $F2 prmu, r_i F_{\ell}(C_{max}, \bar{C})$ problem .....	183
7.1.4 The $F2 prmu \epsilon(\bar{C}/C_{max})$ problem .....	184
7.1.5 The $F2 prmu, d_i \#(C_{max}, T_{max})$ problem .....	189
7.1.6 The $F2 prmu, d_i \#(C_{max}, \bar{U})$ problem .....	192
7.1.7 The $F2 prmu, d_i \#(C_{max}, \bar{T})$ problem .....	194
7.2 $m$ -machine flowshop problems .....	197
7.2.1 The $F prmu Lex(C_{max}, \bar{C})$ problem .....	197
7.2.2 The $F prmu \#(C_{max}, \bar{C})$ problem .....	199
7.2.3 The $F prmu, d_i \epsilon(C_{max}/T_{max})$ problem .....	204
7.2.4 The $F p_{i,j} \in [\underline{p}_{i,j}; \bar{p}_{i,j}], prmu F_{\ell}(C_{max}, \overline{CC}^w)$ problem .....	207
7.2.5 The $F p_{i,j} = p_i \in [\underline{p}_i; \bar{p}_i], prmu \#(C_{max}, \overline{CC}^w)$ problem .....	208

7.2.6	The $F prmu, d_i, nmit F_\ell(\bar{E}^w, \bar{T}^w)$ problem	211
7.3	Jobshop and Openshop problems	213
7.3.1	Jobshop problems	213
7.3.2	The $O2  Lex(C_{max}, \bar{C})$ problem	213
7.3.3	The $O3  Lex(C_{max}, \bar{C})$ problem	215
<b>8.</b>	<b>Parallel machines problems</b>	<b>217</b>
8.1	Problems with identical parallel machines	217
8.1.1	The $P2 pmtn, d_i \in (L_{max}/C_{max})$ problem	217
8.1.2	The $P3 pmtn, d_i \in (L_{max}/C_{max})$ problem	220
8.1.3	The $P2 d_i Lex(T_{max}, \bar{U})$ problem	223
8.1.4	The $P d_i \#(\bar{C}, \bar{U})$ problem	225
8.1.5	The $P pmtn Lex(\bar{C}, C_{max})$ problem	226
8.1.6	The $P d_i = d \text{ non restrictive, } nmit F_\ell(\bar{E}, \bar{T})$ problem	227
8.1.7	The $P d_i = d \text{ unknown, } nmit F_\ell(\bar{E}, \bar{T})$ problem	230
8.1.8	The $P d_i = d \text{ non restrictive, } nmit f_{max}(\bar{E}^w, \bar{T}^w)$ problem	236
8.1.9	The $P d_i = d \text{ unknown, } p_i = p, nmit F_\ell(\bar{E}, \bar{T}, d)$ problem	238
8.2	Problems with uniform parallel machines	241
8.2.1	The $Q p_i = p  \in (f_{max}/g_{max})$ problem	241
8.2.2	The $Q p_i = p  \in (\bar{g}/f_{max})$ problem	247
8.2.3	The $Q pmtn  \in (\bar{C}/C_{max})$ problem	247
8.2.4	The $Q d_i = d \text{ unknown, } nmit F_\ell(\bar{E}, \bar{T})$ problem	253
8.3	Problems with unrelated parallel machines	255
8.3.1	The $R p_{i,j} \in [p_{i,j}, \bar{p}_{i,j}] F_\ell(\bar{C}, \bar{C}^w)$ problem	255
8.3.2	The $R p_{i,j} \in [p_{i,j}, \bar{p}_{i,j}], d_i = d \text{ unknown} F_\ell(\bar{T}, \bar{E}, \bar{C}^w)$ problem	257
8.3.3	The $R pmtn  \in (F_\ell(I_{max}, \bar{M})/C_{max})$ problem	258
<b>9.</b>	<b>Shop problems with assignment</b>	<b>263</b>
9.1	A hybrid flowshop problem with three stages	263
9.2	Hybrid flowshop problems with $k$ stages	264
9.2.1	The $HFK, (PM^{(\ell)})_{\ell=1}^k    F_\ell(C_{max}, \bar{C})$ problem	264
9.2.2	The $HFK, (PM^{(\ell)})_{\ell=1}^k    \in (\bar{C}/C_{max})$ problem	266
9.2.3	The $HFK, (PM^{(\ell)}(t))_{\ell=1}^k  r_i^{(1)}, d_i^{(k)}  \in (C_{max}/T_{max})$ problem	266
<b>A.</b>	<b>Notations</b>	<b>271</b>
A.1	Notation of data and variables	271
A.2	Usual notation of single criterion scheduling problems	271

<b>B. Synthesis on multicriteria scheduling problems</b> .....	277
B.1 Single machine Just-in-Time scheduling problems .....	277
B.2 Single machine problems .....	278
B.3 Shop problems .....	280
B.4 Parallel machines scheduling problems .....	281
B.5 Shop scheduling problems with assignment .....	282
<b>References</b> .....	283
<b>Index</b> .....	301