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The Diversity of Models in Statistical Mechanics: Views about the Structure of Scientific Theories

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1 Introduction

My aim in this paper is to investigate both historically and philosophically some developments of statistical mechanics in order to gain insights into the nature of scientific theories. Picking out examples in the history as well as in contemporary issues, I shall analyze some of the fundamental problems facing statistical mechanics to work out general statements about what scientific theories are. Borrowing formalization methods from the physical sciences themselves as well as from philosophy, I shall attempt at showing that intertheoretical links are as essential to a scientific theory as is its internal hierarchical structure.

What is striking about statistical mechanics is that physicists still disagree on some of its fundamental concepts and principles, though this

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theory was born 140 years ago. For instance, there is no general agreement yet about the definition of the concept of entropy, which is one of the most fundamental concepts in statistical mechanics. Debates are still going on among the leading physicists in the domain¹, and the incompatibility of their various positions leads to puzzling questions about the nature of statistical mechanics².

I shall consider statistical mechanics as a ‘living’ theory, *i.e.*, not as an abstract, intentional entity the immutable content of which would be expressed in textbooks: I shall view it as a general framework of active research and discussions about the very nature of the phenomena and laws which are at its core.

Some philosophers of science, following Stegmüller³, are engaged in a ‘structuralist’ programme aiming at displaying the model-theoretical structure of scientific theories. I will place my contribution at a previous stage of this enterprise. In order to reconstruct the model-theoretical structure of a scientific theory, it is first necessary to identify its main principles and theorems, that is, to have a view of its global hierarchical structure. This global view of the hierarchy of any theory T seems, *prima facie*, not very difficult to catch, because it is precisely that view that corresponds to the common notion one can have of T . For instance, to have a notion of classical mechanics is to know first that its principles are the law of inertia, the law relating force to acceleration, and the law of action and reaction, and second that some of its theorems concern celestial motions. To have a notion of thermodynamics is to know first that its principles are conservation of energy, increase of entropy, and zero of temperatures, and second that some of its theorems concern refrigerators and flows of heat.

The global structure of statistical mechanics seems as easy to catch, in this loose way, as that of classical mechanics or thermodynamics. However, as soon as we try to set out a core principle of statistical mechanics in some precise formulation, we are subject to the objection according to which another, different principle would do the same job. For instance, it can be said that the principles governing the investigation of the microscopic behavior of molecules in statistical mechanics are the laws of motion of classical mechanics; and it can be replied that quantum, not

¹ This was particularly striking during the 1998 STATPHYS Conference.

² For philosophical presentations of these debates, see for instance [Ski93b] and [Gut99].

³ Cf. [Ste179].

classical mechanics is appropriate to provide the laws of motion for the microscopic scale. Or, as another example, some physicists claim that a mathematical property of dynamical systems, ergodicity⁴, is responsible for the apparition of irreversible behavior in macroscopic systems, whereas others reply that mathematical properties like ergodicity or mixing⁵ do not play any causal role in the apparition of irreversibility itself, but only in some temporal properties of irreversible behaviors, like the rate of setting-up of equilibrium.

Accordingly, before being able to disclose the model-theoretical structure of statistical mechanics, it is first necessary to find out a way of characterizing it which would not suffer from any objection of the type I have just alluded to. In order to do so, one primary task is to look for all the different ways within which statistical mechanics is developed, *i.e.*, for all the different kinds of models which have been imagined in the framework of this theory.

The term ‘model’ is known to display a polysemy which is nowhere as problematic as in philosophy of science. Since the development of the semantic view of scientific theories⁶, according to which a theory is not a set of theorems deduced from axioms, but rather a set of models, *i.e.*, structures verifying axioms, the term ‘model’ is used in philosophy of science both in this logical sense and in the more classical sense it has in the sciences, as exemplified in expressions like ‘Bohr model of the atom’, ‘Ising model’, etc. The view commonly held among philosophers of science is that these two uses of the term ‘model’ constitute a case of pure homonymy⁷. It is indeed true that the models in the first, logical, sense are usually of no use to the scientists, being products of the reconstructing activity of the philosophers. Nevertheless, as models in this first sense are mainly mathematical entities, they sometimes resemble models in the second sense, especially when they are built within theoretical domains, like statistical mechanics. The models referred to in the title of this paper are thus mostly models in the second sense, *i.e.*, they are called ‘models’ by the physicists who build them, as well as models in the first sense. However, the fact that they appeal to rather different theoretical principles makes them as difficult to handle from a structuralist point of

⁴ Cf. Sections 2.1 and 3.2.

⁵ The notion of mixing is defined in Section 3.2.

⁶ Cf. for instance [Sup₁67], [Sup₀88], and [vFr80].

⁷ Cf. [Bla62a].