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# Mathematical Logic and Natural Language: Life at the border

**Johan van Benthem**

Institute for Logic Language and Computation (ILLC)
University of Amsterdam
Plantage Muidergracht 24
1018 TV Amsterdam
The Netherlands

Center for the Study of Language and Information (CSLI)
Stanford University
Ventura Hall
Stanford 94305–4115
U.S.A.

E-mail: johan@science.uva.nl, johan@csli.stanford.edu

**Abstract.** This is a brief history of interfaces between logic and language in the 20th century, written by a non-historian. My aim is to show a complex and rich relationship, constantly evolving, which defies easy summaries or generalizations. Moreover, I want to make the reader aware of some grand unifying patterns, that are sometimes lost while peering over type-theoretical formulas, ritualized example sentences, and the general business of exchanging broadsides with one's scientific neighbours.

## 1 Growing up together, and breaking up

Logic and grammar have been close historically, ever since Aristotle laid the formal foundations of both. This conjunction continued into the Middle Ages, with the work of the Scholastics, and one still finds it in the 17th

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century Logic of the Port-Royal. But at some stage in the 19th century, a break occurred. Gottlob Frege's *Begriffsschrift* of 1879 is an uncompromising departure from natural language, in favour of formal languages for logical purposes. To be sure, Frege –living in the Jena of Carl Zeiss– still draws the famous comparison between natural language and formal language as the eye versus the microscope. The former is much less precise, though in return, much more versatile than the latter. But soon after, with Russell, we find the highly critical 'Misleading Form Thesis'. Natural languages obfuscate true meanings by their systematically misleading forms of assertion. In the hands of Carnap and other people close to the Vienna Circle, this became a powerful weapon to be wielded against the philosophical establishment. Accordingly, the study of language and that of logic went separate ways in the first half of the 20th century. Linguistics produced de Saussure, and the subsequent structuralist school. Logic went into its 'mathematical turn', which resulted in the Golden Age with Hilbert, Gödel, Tarski, Turing, and so many others in the amazing twenties and thirties. This period also saw the consolidation of families of formal languages, that textbooks nowadays see as defining the field: first-order predicate logic, second-order logic, lambda calculus and type theories.

Of course, the mathematical turn also meant a shift in agenda. Logic had been a general study of methodology and reasoning in all its varieties in the great works of Bolzano or Mill—it now became a tool for analyzing mathematical proof and foundations of the exact sciences.

## 2 Rapprochement

In the 1950s –another period of great intellectual ferment– the seeds were sown for a new rapprochement. First, there was the undeniable fact that natural language was proving rather resilient. Misleading as it was, it even maintained its position as the favoured medium of communication inside mathematics—where no one who mattered showed any inclination to adopt the more austere medicines prescribed by formal logicians. Inside philosophy, there was even a back-lash in the heartland of analytical philosophy. British 'natural language philosophers' like Ryle started mining the philosophical "gold in them hills", as Austin once phrased it: the systematic patterns in natural language that guide our thinking and practice. Eventually, this work led to such specific achievements as

speech act theory (Searle, Grice) systematizing the various uses that people make of language.

Perhaps too much has been made of the opposition between this style of philosophizing and 'formal language philosophy'. The fact is that, also in what came to be called philosophical logic, natural language was on the rise. This was a period of advances in modal logic (Carnap, Hintikka, Kripke) and temporal logic (Prior) which derived their inspiration from a mix of linguistic observations about various modal expressions and verb tenses, and rigorous mathematical technique. All these things were not yet full-fledged collaborations between logic and linguistics, but they did fill Russell's gap. Indeed, some logicians in this period explicitly advocated natural language as an 'area of application' for mathematical logic—thinking in a somewhat 'colonizing mode'. Examples are Reichenbach, whose work on temporal expressions is still cited today, and Curry, who saw a new scope for his formal language systems, such as combinatory logic, in these broader fields. Finally, but this is only with hindsight, the 1950s also produced other trends that are highly relevant to logic-language interfaces as conceived today, such as the advent of game theory, and its penetration –up to a point– into logic and philosophy.

In the Netherlands, the 1950s are seen as a stagnant decade of the last century, where people devoted themselves to what is now seen as the boring pursuit of happiness inside their own families. The more one thinks about what actually happened scientifically and economically, the shallower this prejudice becomes.

## 3 Mathematical linguistics

Modern linguistics started with the work of Harris and his student Chomsky, whose Syntactic Structures (1957) revolutionized the field. It gave an exact analysis of grammars for natural languages, derived from inspirations in mathematical logic, which showed that, at least in principle, natural language has firm structures that can be studied by formal means. Incidentally, Chomsky's book was submitted by the logician Beth for publication in the well-known series *Studies in Logic and the Foundations of Mathematics*—and rejected by Heyting. Prophecy was not a gift of the intuitionists, inspired though they were otherwise. In the 1960s, this work generated the first significant interaction between mathematics and linguistics, which still exists today: the theory of formal languages