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Selected Solutions of Einstein’s Field Equations: Their Role in General Relativity and Astrophysics

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