Preface

In many areas of theoretical physics, geometrical and functional analytic methods are used simultaneously. During the course of our studies and education, these methods appeared to be extremely different. For classical field theories, a differential complex over smooth manifolds, smooth sections of vector bundles over manifolds, principal fibre bundles, connections, curvatures, etc. are introduced. For quantum field theory, singular operator-valued distributions act on Fock space and lead to all the well-known difficulties. After the work of Alain Connes and the many applications of non-commutative geometry to various subjects, we became familiar with, for example, the fact that on any associative superalgebra at least one differential calculus exists. But clearly there are many more aspects. We selected seven subjects and lecturers to cover symplectic geometry, quantum gravity, string theory, an introduction to non-commutative geometry, and the application to fundamental interactions, to the quantum Hall effect as well as to physics on q-deformed spaces.

In the first contribution, Anton Alexseev uses techniques of symplectic geometry to evaluate certain integrals over manifolds that have special symmetries. For them the stationary phase approximation becomes exact. Starting from simple examples, he introduces equivariant cohomology and sketches the proof of localization formulas of Duistermaat - Heckman type. The Weil model, well-known from the BRST formalism, is introduced, and a non-commutative generalization is used to prove group-valued localization.

John Baez elucidates the present status of quantum geometry of space-time. Since the implementation of constraints within the Hamiltonian approach to Einstein gravity is complicated, he explains in detail the simplifying BF system connected to the Chern - Simons model. We learn about spin net works and spin foams, become familiar with triangulations of four-manifolds, and calculate spectra of quantum tetrahedra. His lecture notes not only survey the subject, but also give an extensive list of references arranged according to nine different subjects.

Cesar Gomez then gives an introduction to string theory. Starting from the mode expansion he explains T-duality, a symmetry relating closed strings of radius R and $1/R$. Next D-brane ideas are represented.

Both quantum gravity and string theory have their own way of treating space-time. But there is a further approach that goes by the name of non-commutative geometry. This is the subject of the chapter by Daniel Kastler.

John Madore introduces a differential complex over an arbitrary associative algebra. We learn that, depending on the procedure, a regularization effect may result. Attempts to add a gravitational field are also reviewed.

This introduction is useful to follow the lectures of Daniel Kastler. He spoke about Connes' approach to the standard model as well as recent ideas for including gravity as an external field. He presents the interesting idea that additional dimensions finally lead via the Higgs effect to masses of particles, as well as the recent attempts to use $SU_q(2)$ for q being a third root of unity to "deduce" the gauge group of the standard model.

Julius Wess applies ideas of non-commutative geometry to the q-deformed Heisenberg algebra. The spectra of position and momentum are discrete. Phase space gets a lattice-like structure. Higher-dimensional analogues are investigated too.

Finally, Ruedi Seiler explains geometrical properties of transport in quantum Hall systems. The integer effect is treated in detail.

Although we learn here about many different applications of non-commutative geometry, the hope is that a unique picture of a quantum space-time may finally result. Quantum theory changes our ideas on geometry, and further surprises may come along soon.

We have tried to cover many subjects and were lucky to be supported by so many excellent lecturers. We hope that the school helped young people seeking an entry to these subjects. They will hopefully remember the excellent atmosphere of the Schladming Winter School 1999.

At this point we also want to express our thanks to the main sponsor of the School, the Austrian Ministry for Science and Transportation. In addition, we grateful acknowledge the generous support by the Government of Styria and the Town of Schladming. Valuable help towards the organization was received from the Wirtschaftskammer Steiermark (Sektion Industrie), Steyr-Daimler-Puch AG, Ricoh-Austria, and Styria Online.

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