Table of Contents

Preface V

Preface to the Second Edition VIII

Part 1. Artificial Randomness

1.0. Survey 3

1.1. Random Generators 4
1.1.1 The Necessity of Sample Values 4
1.1.2 Random Numbers 5
1.1.3 Obtaining Random Numbers 5
1.1.4 Requirements 6
1.1.5 Pseudo Random Generators 6
1.1.5.1 Von Neumann Generator 7
1.1.5.2 Quadratic Residue Generator 7
1.1.5.3 Linear Congruential Generator 7
1.1.5.4 Tausworthe Generator 7
1.1.6 Linear Congruential Generators 8
1.1.7 Tausworthe Generators and Galois Fields 9
1.1.8 Tausworthe Generators: T-Machine 11

1.2. Our Collection of Random Generators 13
1.2.1 About our Collection 13
1.2.2 TurboPascal 13
1.2.3 Von Neumann 14
1.2.4 Multi 15
1.2.5 Linear1 18
1.2.6 Linear2 18
1.2.7 Quadres 19
1.2.8 Tausworthe1 20
1.2.9 Tausworthe2 21
1.2.10 Radio 23

1.3. Tests for Random Generators 24
1.3.1 Statistical Tests 24
1.3.1.1 χ^2 Goodness-of-fit Test 24
1.3.1.2 Classic Tests 27
1.3.1.3 Variable Sample Sizes 39
1.3.2 Time Series Analysis Tests 41
1.3.2.1 Methods of Time Series Analysis 41
1.3.2.2 The Autocovariance Function 42
1.3.2.3 Asymptotic Behavior of the Autocorrelation Coefficients 44
1.3.2.4 Fourier Transforms 44
1.3.2.5 The Periodogram 45
1.3.2.6 Computer Experiment 47
1.3.2.7 Results 49
1.3.3 Visual Tests 50
1.3.3.1 Uniform Generation 50
1.3.3.2 Geometric Structures 51
1.3.3.3 Patterns 54

1.4. Generating Sample Values 58
1.4.1 Generating Concepts 58
1.4.1.1 Transformation with $\hat{F}$ 58
1.4.1.2 Convolution Operations 63
1.4.2 $N(0,1)$ Sample Values 64
1.4.2.1 Box–Mueller Method 64
1.4.2.2 Polar–Marsaglia Method 67
1.4.2.3 Central Limit Theorem 71
1.4.3 Sample Values of Various Distributions 72
1.4.3.1 Binomial Distribution 72
1.4.3.2 Poisson Distribution 73
1.4.3.3 Geometric Distribution 75
1.4.3.4 Exponential Distribution 76
1.4.3.5 Fermi-Dirac Distribution 77
1.4.3.6 Erlang Distribution 78
1.4.3.7 χ^2 Distribution 79

1.5. Goodness-of-fit Tests for Arbitrary Distributions 80
1.5.1 Fundamentals 80
1.5.2 Computer Experiment 80
1.5.3 Test Results 81

1.6. Monte-Carlo Integration 83
1.6.1 Estimation of a Definite Integral 83
1.6.2 Hit-or-Miss and Mean-Value Methods 83
1.6.2.1 Hit-or-Miss Method 83

1.6.2.2 Mean-Value Method 84
1.6.3 Computer Experiment 85

1.7. Bibliographical Comments 86
1.7.1 Random Generators 86
1.7.2 Tests for Random Generators 87
1.7.3 Generating Sample Values 87
1.7.4 Monte-Carlo Integration 88

Part 2. Stochastic Models

2.0. Survey 91

2.1. Buffon Experiments 93
2.1.1 The Buffon Problem 93
2.1.2 Various Experiments 93
2.1.3 Computer Experiments 94

2.2. Service Systems 95
2.2.1 Comparison of Two Service Modes 95
2.2.1.1 Waiting System 95
2.2.1.2 Service Modes 95
2.2.1.3 Performance of Service Systems 95
2.2.1.4 Model Description 96
2.2.1.5 Computer Experiment 98
2.2.2 Programs 98
2.2.2.1 Program "WaitingSystem" 98
2.2.2.2 Main Data Structure 100
2.2.2.3 Procedure `RunThrough` 100
2.2.2.4 The Elements of the Procedure `RunThrough` 100

2.3. Kinetic Gas Theory 102
2.3.1 Ideal Gas 102
2.3.1.1 Contributions of Maxwell and Boltzmann 102
2.3.1.2 Momentum Absorption 102
2.3.1.3 Pressure 103
2.3.1.4 Stochastic Model 103
2.3.1.5 Temperature 105
2.3.1.6 State Equation 105
2.3.1.7 Computer Experiment 106
2.3.2 Programs 106
2.3.2.1 Program "IdealGas" 106
2.3.2.2 The Main Data Structure 107
2.3.2.3 Procedure `Init` 107
2.3.2.4 Procedure `RunThrough` 108

2.3.2.5 The Elements of the Procedure `RunThrough` 109
2.3.2.6 Procedure `Evaluation` 109

2.4. Kinetic Dynamics and Equilibrium 110
2.4.1 The Contributions of Maxwell and Boltzmann as a Starting Point 110
2.4.2 Kinetic Dynamics 110
2.4.3 The Kernel Density Estimator 112
2.4.4 Computer Experiment 113
2.4.5 Asymptotics 114
2.4.6 Video Clips 115

2.5. Bibliographical Comments 117
2.5.1 Buffon Experiments 117
2.5.2 Service Systems 117
2.5.3 Kinetic Gas Theory 118
2.5.4 Kinetic Dynamics and Equilibrium 118

Part 3. Stochastic Processes

3.0. Survey 121

3.1. Markov Chains 123
3.1.1 Conditioning on the Current State 123
3.1.1.1 Markov Property 123
3.1.1.2 The Markov Chain 123
3.1.1.3 Computer Experiment 125
3.1.2 An Application 125
3.1.2.1 Typical Applications 125
3.1.2.2 The Problem 125
3.1.2.3 Sketch of the Proof 126
3.1.2.4 Computer Experiment 126
3.1.3 Programs 127
3.1.3.1 The Concept 127
3.1.3.2 Description 128

3.2. Birth and Death Processes 130
3.2.1 Evaluating Trajectories 130
3.2.1.1 The Simple Birth and Death Process 130
3.2.1.2 Mean and Extinction Probability 130
3.2.1.3 The Chapman–Kolmogorov Equation 131
3.2.1.4 Computer Experiment 132
3.2.2 Programs 132
3.2.2.1 Generating Trajectories 132

3.3. Diffusion Processes 134
3.3.1 Brownian Motion 134
3.3.1.1 The Physical Phenomenon 134
3.3.1.2 The Stochastic Process 134
3.3.1.3 Computer Experiment 135
3.3.2 Dirichlet Problem 136
3.3.2.1 A Physical Problem 136
3.3.2.2 Mathematical Formulation 136
3.3.2.3 Sketch of the Proof 137
3.3.2.4 Computer Experiment 138
3.3.3 Programs 140
3.3.3.1 Generating Trajectories 140
3.3.3.2 The Iteration 140
3.3.3.3 Data Structures 140

3.4. Control of Traffic Lights 143
3.4.1 The Green Time as Control Variable 143
3.4.1.1 The Technical Model 143
3.4.1.2 Volume of Traffic 144
3.4.1.3 Computer Experiment 145
3.4.2 The Critical Green Time 146
3.4.2.1 Traffic Flow 146
3.4.2.2 The Queueing Process 146
3.4.2.3 Iteration of Distributions 147
3.4.2.4 Discussion of the Model 148
3.4.2.5 Collapse of Traffic 148
3.4.2.6 The Critical Green Time 149
3.4.2.7 Computer Experiment 149
3.4.3 Ergodicity 150
3.4.3.1 Independence from the Initial Distribution 150
3.4.3.2 Proving Methods 150
3.4.3.3 Computer Experiment 151
3.4.4 Optimal Green Times 151
3.4.4.1 The Concept of Optimal Green Time 151
3.4.4.2 Numerical Analysis 152
3.4.4.3 RiLSA 152
3.4.4.4 Computer Experiment 153
3.4.5 Computer Validation 154
3.4.5.1 Objective 154
3.4.5.2 Computer Experiment 154

3.5. Bibliographical Comments 156
3.5.1 Markov Processes 156
3.5.2 Birth and Death Processes 156
3.5.3 Diffusion Processes 156
3.5.4 Control of Traffic Lights 157

Part 4. Evaluation of Statistical Procedures

4.0. Survey 161

4.1. Experimental Evaluation 163
4.1.1 What is Statistics? 163
4.1.2 Empirical Evaluation and Experimentation 164
4.1.3 Evaluation 164

4.2. Neyman Pearson Lemma 166
4.2.1 Test Problem 166
4.2.2 The Solution Concept 167
4.2.3 The Fundamental Lemma 167
4.2.4 Computer Experiment 169

4.3. Wald Sequential Test 172
4.3.1 Sequential Probability Ratio Test 172
4.3.2 Test Procedure 172
4.3.3 Operations Characteristic 174
4.3.4 Computer Experiment 175

4.4. Bayesian Point Estimation 177
4.4.1 Bayesian Model 177
4.4.2 Bayesian Concept 178
4.4.3 A Posteriori Distribution 178
4.4.4 Various Loss Functions 179
4.4.5 Computer Experiment 180

4.5. Hartigan Procedures 183
4.5.1 Description 183
4.5.1.1 Hartigan Procedure 183
4.5.1.2 The Subsample Method 184
4.5.1.3 Computer Experiment 185
4.5.2 Computer Program 186
4.5.2.1 Program Description 186
4.5.2.2 Preliminary Considerations for the Subsample Method 186
4.5.2.3 Problems with Storage Capacity During the Subsample Method 188

4.6. Bibliographical Comments 191
4.6.1 Experimental Evaluation 191
4.6.2 Neyman Pearson Lemma 191
4.6.3 Wald Sequential Test 191
4.6.4 Bayesian Point Estimation 192
4.6.5 Hartigan Procedures 192

References 195

Symbols 201

Index 205