Preface

All necessary reasoning is strictly speaking mathematic reasoning, that is to say, it is performed by observing something equivalent to a mathematical diagram.

Charles Sanders Peirce

The quotation of Peirce is taken from his 1898 Cambridge lectures: Reasoning and the Logic of Things. Today – more than 100 years later – this title is still of striking topicality. An enormous variety of scientists, from fields such as philosophy, mathematics, computer science, linguistics and biology, devote themselves to research on human knowledge and thinking. Logic, however, which was understood as the theory of the forms of thinking in centuries past, is nowadays understood as *mathematical* logic in a much narrower sense, and has lost its former connection to the so-called *elementary* logic to a large degree.

In his Contextual Logic, Rudolf Wille makes an attempt to take the former understanding of logic into consideration again and to endow it with the same mathematical preciseness that marks mathematical logic. As he says in [77], '*Contextual Logic*' is a kind of logic which is '*grounded on the traditional philosophical understanding of logic as the doctrine of the forms of thinking*' and aims to support knowledge representation and knowledge processing.

Since the 16th century, human knowledge and thinking have been understood to be made up of three parts: concepts, judgements, and conclusions. Concepts are the basic units of thinking, judgements are combinations of concepts and facts, and conclusions are entailments between judgements. Thus, elementary logic was presented in three parts as well, namely the doctrine of concepts, the doctrine of judgements, and the doctrine of conclusions.

Contextual logic is based on a mathematization of these doctrines. Thus, Contextual logic has to provide mathematizations for concepts, judgements and conclusions. Concepts have already been formalized in Formal Concept Analysis (FCA). They are units of thought which are described by their extension and intension. These two components are formalized in FCA. A formal concept is a pair of sets: its extension, which is a set of (formal) objects and is called the *extent of the formal concept*, and its intension, which is a set of (formal) attributes and is called the *intent of the formal concept*. The relationship between the extent and the intent of a concept is as follows: In the intent of a formal concept, we find exactly all attributes which apply to all objects of the extent, and, vice versa, in the extent of the formal concept we find exactly all objects which satisfy all attributes of the intent (for the mathematical details we refer the reader to [19]). FCA was introduced by Wille in 1981, and since then it has been used successfully in more than 200 projects.

An approach for a formalization of judgements and conclusions can be found in Sowa's theory of conceptual graphs (see [59], [65]). Conceptual graphs are based on the existential graphs of Peirce and are developed to express meaning that is humanly readable and understandable as well as precise and computationally tractable. Conceptual graphs can be understood as formal judgements that are closely related to natural language. In particular, they are graphs that consist of concept nodes, which bear references as well as types of the references. The concept boxes are connected by edges, which are used to express different relationships between the referents of the attached concept boxes. Sowa provides rules for formal deduction procedures on conceptual graphs; hence the system of conceptual graphs offers a formalization of conclusions too.

As FCA provides a formalization of concepts, and as conceptual graphs offer a formalization of judgements and conclusions, a convincing idea is to combine these approaches to gain a unified formal theory for concepts, judgements and conclusions, i.e., a formal theory of elementary logic. In [75], Wille marked the starting point for a such a theory. There he provided a mathematization of conceptual graphs where the types of conceptual graphs are interpreted by formal concepts of a so-called power context family. The resulting graphs are called *concept graphs*. They form the mathematical basis for contextual logic.

In her Ph.D.-thesis 'Kontextuelle Urteilslogik mit Begriffsgraphen' ('Contextual Judgement Logic with Concept Graphs,' see [44]) Prediger worked out the theory of concept graphs using the foundation of Wille's ideas. In contrast to Wille, she separated the syntax and semantics of concept graphs. However, for the semantics she adopted the contextual semantics, i.e., the power context families, of Wille. Her syntax allows us to express existential quantification (with the so-called *generic marker*), and the description of situations and contexts (she allowed the *nestings of graphs*). Furthermore, she provided a sound and complete calculus. Therefore, the structure of her thesis can be outlined by the keywords syntax, semantics and calculus; hence it is closely related to the way mathematical logic is treated. But it has to be stressed that the *intention* of her thesis was different: The one-sided extensional view of mathematical logic and its focus on the truth values of formulas is refused, but her thesis is in line with contextual logic, i.e., a formalization of concepts, judgements and conclusions.

In Prediger's elaboration of concept graphs it is not possible to express negations, so this is the next step in the further development of concept graphs. Negations can occur on different levels: It should be possible to express the negation of concepts or relations, and it should be possible to express that judgements have the form of negations.

The negation of concepts is a difficult philosophical problem, so it is hard to implement a formal negation of concepts in FCA and concept graphs. In fact, there are at least two different possibilities for implementing negation on the formal concepts, namely on the side of their extents and on the side of their intents. The term 'negation' is used for a negation on the extensional side, for the intensional side the term 'opposition' is introduced (for this and the rest of this paragraph, see [77]). The next problem one has to face is that the negation or opposition of a formal concept is not a formal concept in general. So, in order to introduce negation on the conceptual level, the definition of formal concepts has to be generalized. This leads to the notion of so-called *semi-concepts* and *proto-concepts*.

In her thesis [29] (see also [28, 30]), Klinger has worked out the syntax and semantics for a negation on the conceptual level which is based on semi-concepts. Implementing negation on the basis of proto-concepts is investigated by Wille in [79] and [80].

In this treatise, how a negation on the level of judgements can be implemented will be elaborated. The main ideas that are needed can be found in the publications of Peirce (for existential graphs) and Sowa (for conceptual graphs). But a mathematical formalization is missing in these publications. Thus, in the tradition of the publications of Wille and Prediger, we provide a mathematization of conceptual graphs in this treatise, where an additional element of existential graphs, the so-called *cuts*[1], is added. Cuts allow us to express negation on the level of judgements. The resulting graphs will be called *concept graphs with cuts*. These graphs will allow us to express existential quantification too (thus, as the universal quantifier can be expressed with the existential quantifier and negations, universal quantification can be expressed as well), but we will not consider nested graphs. Similar to the thesis of Prediger, we will distinguish between syntax and semantics, and we will also provide a sound and complete calculus. Thus, in a sense, this treatise can be seen as a continuation of Prediger's thesis.

[1] The term *cut* is adopted from Peirce, where it denotes a syntactical device of existential graphs which is used to express negation. This term should not be mixed up with the term *cut* as used in mathematical logic.

In fact, as we can express relations between objects, conjunction and negation in judgements, and existential quantification, it will turn out that concept graphs with cuts are equivalent to first-order predicate logic. More precisely, it is the goal of this treatise to work out and mathematize a fragment of conceptual graphs which corresponds to first-order predicate logic. However, concept graphs with cuts are grounded on a contextual understanding of logic and are designed to fit in the framework of contextual logic. Hence, this treatise can show that '*Contextual Logic may reach at least the expressibility of first order predicate logic*' (see [77]).

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