
Contents

Preface	IX
1 Arrays of Point and Line Sources, and Optimization	1
1.1 The Problem of Antenna Optimization	1
1.2 Arrays of Point Sources	2
1.2.1 The Linear Array	3
1.2.2 Circular Arrays	10
1.3 Maximization of Directivity and Super-gain	15
1.3.1 Directivity and Other Measures of Performance	15
1.3.2 Maximization of Directivity	19
1.4 Dolph-Tschebysheff Arrays	21
1.4.1 Tschebysheff Polynomials	22
1.4.2 The Dolph Problem	24
1.5 Line Sources	26
1.5.1 The Linear Line Source	30
1.5.2 The Circular Line Source	36
1.5.3 Numerical Quadrature	43
1.6 Conclusion	47
2 Discussion of Maxwell's Equations	49
2.1 Introduction	49
2.2 Geometry of the Radiating Structure	49
2.3 Maxwell's Equations in Integral Form	50
2.4 The Constitutive Relations	51
2.5 Maxwell's Equations in Differential Form	52
2.6 Energy Flow and the Poynting Vector	55
2.7 Time Harmonic Fields	56
2.8 Vector Potentials	58
2.9 Radiation Condition, Far Field Pattern	60
2.10 Radiating Dipoles and Line Sources	63
2.11 Boundary Conditions on Interfaces	68

VI Contents

2.12 Hertz Potentials and Classes of Solutions	70
2.13 Radiation Problems in Two Dimensions	73
3 Optimization Theory for Antennas	77
3.1 Introductory Remarks.....	77
3.2 The General Optimization Problem	80
3.2.1 Existence and Uniqueness	81
3.2.2 The Modeling of Constraints	84
3.2.3 Extreme Points and Optimal Solutions	88
3.2.4 The Lagrange Multiplier Rule.....	93
3.2.5 Methods of Finite Dimensional Approximation	96
3.3 Far Field Patterns and Far Field Operators	101
3.4 Measures of Antenna Performance	103
4 The Synthesis Problem	113
4.1 Introductory Remarks.....	113
4.2 Remarks on Ill-Posed Problems	115
4.3 Regularization by Constraints	121
4.4 The Tikhonov Regularization	127
4.5 The Synthesis Problem for the Finite Linear Line Source	133
4.5.1 Basic Equations	134
4.5.2 The Nyström Method	135
4.5.3 Numerical Solution of the Normal Equations	137
4.5.4 Applications of the Regularization Techniques.....	138
5 Boundary Value Problems for the Two-Dimensional Helmholtz Equation	145
5.1 Introduction and Formulation of the Problems	145
5.2 Rellich's Lemma and Uniqueness	148
5.3 Existence by the Boundary Integral Equation Method	151
5.4 L^2 -Boundary Data.....	157
5.5 Numerical Methods	163
5.5.1 Nyström's Method for Periodic Weakly Singular Kernels	164
5.5.2 Complete Families of Solutions	168
5.5.3 Finite Element Methods for Absorbing Boundary Conditions	174
5.5.4 Hybrid Methods.....	181
6 Boundary Value Problems for Maxwell's Equations	185
6.1 Introduction and Formulation of the Problem	185
6.2 Uniqueness and Existence	188
6.3 L^2 -Boundary Data.....	193

7 Some Particular Optimization Problems	195
7.1 General Assumptions	195
7.2 Maximization of Power	197
7.2.1 Input Power Constraints	198
7.2.2 Pointwise Constraints on Inputs	202
7.2.3 Numerical Simulations	204
7.3 The Null-Placement Problem	211
7.3.1 Maximization of Power with Prescribed Nulls	213
7.3.2 A Particular Example – The Line Source	216
7.3.3 Pointwise Constraints	219
7.3.4 Minimization of Pattern Perturbation	221
7.4 The Optimization of Signal-to-Noise Ratio and Directivity	226
7.4.1 The Existence of Optimal Solutions	227
7.4.2 Necessary Conditions	228
7.4.3 The Finite Dimensional Problems	231
8 Conflicting Objectives: The Vector Optimization Problem	239
8.1 Introduction	239
8.2 General Multi-criteria Optimization Problems	240
8.2.1 Minimal Elements and Pareto Points	241
8.2.2 The Lagrange Multiplier Rule	247
8.2.3 Scalarization	249
8.3 The Multi-criteria Dolph Problem for Arrays	250
8.3.1 The Weak Dolph Problem	251
8.3.2 Two Multi-criteria Versions	253
8.4 Null Placement Problems and Super-gain	262
8.4.1 Minimal Pattern Deviation	264
8.4.2 Power and Super-gain	270
8.5 The Signal-to-noise Ratio Problem	278
8.5.1 Formulation of the Problem and Existence of Pareto Points	278
8.5.2 The Lagrange Multiplier Rule	280
8.5.3 An Example	282
A Appendix	285
A.1 Introduction	285
A.2 Basic Notions and Examples	286
A.3 The Lebesgue Integral and Function Spaces	292
A.3.1 The Lebesgue Integral	292
A.3.2 Sobolev Spaces	295
A.4 Orthonormal Systems	298
A.5 Linear Bounded and Compact Operators	300
A.6 The Hahn-Banach Theorem	307
A.7 The Fréchet Derivative	310
A.8 Weak Convergence	312

VIII Contents

A.9 Partial Orderings	315
References	319
Index	327