

Contents

Preface	ix
1 Measure and Integral	1
1.1 Measure	1
1.2 Borel Set	6
1.3 Lebesgue Measure	7
1.4 Measurable Function	8
1.5 Lebesgue Integral	13
1.6 Lebesgue-Stieltjes Integral	17
2 Probability Theory	21
2.1 Three Axioms	21
2.2 Random Variables	25
2.3 Probability Distribution	31
2.4 Independent and Identical Distribution	36
2.5 Expected Value Operator	40
2.6 Variance, Covariance and Moments	53
2.7 Optimistic and Pessimistic Values	54
2.8 Some Inequalities	56
2.9 Characteristic Function	59
2.10 Convergence Concepts	61
2.11 Laws of Large Numbers	65
2.12 Conditional Probability	71
2.13 Stochastic Simulations	73
3 Credibility Theory	79
3.1 Four Axioms	80
3.2 Fuzzy Variables	87
3.3 Credibility Distribution	95
3.4 Independent and Identical Distribution	103
3.5 Optimistic and Pessimistic Values	107
3.6 Expected Value Operator	109
3.7 Variance, Covariance and Moments	124
3.8 Some Inequalities	125
3.9 Characteristic Function	127

3.10	Convergence Concepts	129
3.11	Fuzzy Simulations	133
4	Trust Theory	137
4.1	Rough Set	137
4.2	Four Axioms	138
4.3	Rough Variable	142
4.4	Trust Distribution	148
4.5	Independent and Identical Distribution	154
4.6	Expected Value Operator	157
4.7	Variance, Covariance and Moments	169
4.8	Optimistic and Pessimistic Values	171
4.9	Some Inequalities	173
4.10	Characteristic Function	175
4.11	Convergence Concepts	177
4.12	Laws of Large Numbers	181
4.13	Conditional Trust	185
4.14	Rough Simulations	188
5	Fuzzy Random Theory	191
5.1	Fuzzy Random Variables	191
5.2	Chance Measure	194
5.3	Chance Distribution	198
5.4	Independent and Identical Distribution	202
5.5	Expected Value Operator	204
5.6	Variance, Covariance and Moments	206
5.7	Optimistic and Pessimistic Values	207
5.8	Convergence Concepts	210
5.9	Laws of Large Numbers	211
5.10	Fuzzy Random Simulations	212
6	Random Fuzzy Theory	215
6.1	Random Fuzzy Variables	215
6.2	Chance Measure	218
6.3	Chance Distribution	223
6.4	Independent and Identical Distribution	226
6.5	Expected Value Operator	227
6.6	Variance, Covariance and Moments	228
6.7	Optimistic and Pessimistic Values	229
6.8	Convergence Concepts	232
6.9	Random Fuzzy Simulations	241
7	Bifuzzy Theory	245
7.1	Bifuzzy Variables	245
7.2	Chance Measure	247
7.3	Chance Distribution	252
7.4	Independent and Identical Distribution	255

7.5	Expected Value Operator	256
7.6	Variance, Covariance and Moments	257
7.7	Optimistic and Pessimistic Values	258
7.8	Convergence Concepts	261
7.9	Bifuzzy Simulations	270
8	Birandom Theory	273
8.1	Birandom Variables	273
8.2	Chance Measure	276
8.3	Chance Distribution	279
8.4	Independent and Identical Distribution	282
8.5	Expected Value Operator	284
8.6	Variance, Covariance and Moments	285
8.7	Optimistic and Pessimistic Values	286
8.8	Convergence Concepts	288
8.9	Laws of Large Numbers	289
8.10	Birandom Simulations	290
9	Rough Random Theory	293
9.1	Rough Random Variables	293
9.2	Chance Measure	296
9.3	Chance Distribution	298
9.4	Independent and Identical Distribution	301
9.5	Expected Value Operator	303
9.6	Variance, Covariance and Moments	303
9.7	Optimistic and Pessimistic Values	305
9.8	Convergence Concepts	307
9.9	Laws of Large Numbers	307
9.10	Rough Random Simulations	308
10	Rough Fuzzy Theory	311
10.1	Rough Fuzzy Variables	311
10.2	Chance Measure	312
10.3	Chance Distribution	316
10.4	Independent and Identical Distribution	319
10.5	Expected Value Operator	320
10.6	Variance, Covariance and Moments	320
10.7	Optimistic and Pessimistic Values	322
10.8	Convergence Concepts	324
10.9	Rough Fuzzy Simulations	327
11	Random Rough Theory	331
11.1	Random Rough Variables	331
11.2	Chance Measure	333
11.3	Chance Distribution	336
11.4	Independent and Identical Distribution	339
11.5	Expected Value Operator	341

11.6	Variance, Covariance and Moments	341
11.7	Optimistic and Pessimistic Values	342
11.8	Convergence Concepts	344
11.9	Laws of Large Numbers	345
11.10	Random Rough Simulations	346
12	Fuzzy Rough Theory	349
12.1	Fuzzy Rough Variables	349
12.2	Chance Measure	351
12.3	Chance Distribution	354
12.4	Independent and Identical Distribution	357
12.5	Expected Value Operator	359
12.6	Variance, Covariance and Moments	360
12.7	Optimistic and Pessimistic Values	361
12.8	Convergence Concepts	364
12.9	Laws of Large Numbers	364
12.10	Fuzzy Rough Simulations	365
13	Birough Theory	369
13.1	Birough Variables	369
13.2	Chance Measure	371
13.3	Chance Distribution	374
13.4	Independent and Identical Distribution	377
13.5	Expected Value Operator	379
13.6	Variance, Covariance and Moments	379
13.7	Optimistic and Pessimistic Values	380
13.8	Convergence Concepts	382
13.9	Laws of Large Numbers	383
13.10	Birough Simulations	384
14	Some Remarks	387
14.1	Uncertainty Theory Tree	387
14.2	Multifold Uncertainty	388
14.3	Ranking Uncertain Variables	388
14.4	Nonclassical Credibility Theory	389
14.5	Generalized Trust Theory	396
	Bibliography	399
	List of Frequently Used Symbols	408
	Index	409