

# Contents

<b>1. Introduction</b> .....	1
<b>2. Overview</b> .....	7
2.1 $k$ -Mass .....	10
2.2 Electrons in Metals .....	12
2.2.1 Jellium Model .....	13
2.3 Electron Fluctuations (Plasmons) .....	16
2.4 Electron-Plasmon Coupling .....	16
2.5 $\omega$ -Mass .....	20
2.6 Total Effective Mass .....	21
2.7 Induced Interaction .....	21
2.8 Examples .....	23
2.9 Minimal Mean Field Theory .....	23
2.9.1 Non-locality in Space of the Exchange Contribution ...	24
2.9.2 Non-locality in Time of the Correlation Contribution .	26
2.10 Phonons .....	26
2.11 Appendix .....	28
2.11.1 Second Quantization .....	28
2.11.2 Single-Particle Self-Energy .....	32
<b>3. Electronic Structure</b> .....	35
3.1 Born-Oppenheimer Approximation .....	35
3.2 Density Functional Theory: Generalities .....	38
3.3 The Local Density Approximation .....	41
3.3.1 Discussion of Kohn-Sham Method .....	43
3.3.2 Pseudopotentials .....	44
3.4 Kohn-Sham Equations in a Spherical Basis .....	45
3.4.1 The Expansion of $V_{\text{ext}}$ in Spherical Harmonics .....	47
3.4.2 The Expansion of $\varrho(\mathbf{r})$ , $v_{\text{H}}$ and $v_{\text{xc}}$ .....	50
3.5 Kohn-Sham Equations in a Cylindrical Basis .....	51
3.6 Appendices .....	54
3.6.1 Basics on Pseudopotentials .....	54
3.6.2 Matrix Elements in a Spherical Basis .....	57

<b>4. Electronic Response to Time-Dependent Perturbations . . .</b>	<b>61</b>
4.1 Linear Response: RPA and TDLDA . . . . .	61
4.2 Strength Function and Sum Rules . . . . .	65
4.3 Transition Density and Plasmons . . . . .	68
4.4 Appendices . . . . .	70
4.4.1 Linear Response . . . . .	70
4.4.2 TDLDA or RPA in the Configuration Space . . . . .	72
4.4.3 Plasmon Wavefunction Components for a Separable Interaction . . . . .	73
4.4.4 Spherical TDLDA . . . . .	74
4.4.5 Cylindrical TDLDA . . . . .	76
<b>5. Applications to Carbon Structures and Metal Clusters . . .</b>	<b>79</b>
5.1 C <sub>60</sub> Fullerene . . . . .	79
5.1.1 Kohn–Sham States for C <sub>60</sub> . . . . .	79
5.1.2 The Optical Absorption of C <sub>60</sub> . . . . .	81
5.2 Carbon Nanotubes . . . . .	85
5.2.1 Electronic Structure . . . . .	86
5.2.2 Electromagnetic Response and Static Polarizabilities . .	87
5.3 Linear Carbon Chains . . . . .	89
5.3.1 Electronic Structure . . . . .	89
5.3.2 Optical Response and Plasmon Resonances . . . . .	93
5.4 Metal Clusters: the Case of Na <sub>8</sub> . . . . .	98
5.5 Appendices . . . . .	100
5.5.1 Role of Symmetries . . . . .	100
<b>6. Phonons: Harmonic Approximation . . . . .</b>	<b>105</b>
6.1 Harmonic Approximation . . . . .	106
6.1.1 A Practical Example: a Linear Triatomic Molecule . .	111
6.2 Phonon Calculations . . . . .	112
6.2.1 Introduction . . . . .	112
6.2.2 Bond Charge Models . . . . .	114
6.2.3 Phonons from ab initio Calculations . . . . .	128
<b>7. The Car–Parrinello Method . . . . .</b>	<b>131</b>
7.1 An Efficient Tool for Solving the Electronic Problem . . . . .	131
7.2 Car–Parrinello Molecular Dynamics . . . . .	133
7.3 Ionic Motion: Parameter-Free Molecular Dynamics Simulation	136
7.3.1 Frozen-Phonon Calculations . . . . .	141
<b>8. Coupling of Electrons to Phonons and to Plasmons . . . . .</b>	<b>145</b>
8.1 Definition of the Electron–Phonon Interaction . . . . .	146
8.2 Diagonal Matrix Elements . . . . .	148
8.3 Electron–Phonon Coupling in C <sub>60</sub> <sup>-</sup> . . . . .	149

8.4	Signatures of the Electron–Phonon Interaction: Photoemission in $C_{60}^-$ .....	154
8.4.1	Electron–Phonon Coupling in Other Fullerenes .....	160
8.4.2	Electron–Phonon Coupling in Sodium Clusters .....	163
8.5	The Effective Electron–Electron Interaction .....	165
8.6	Coupling of Electrons to Plasmons .....	166
8.7	TDLDA with Electron–Plasmon and Electron–Phonon Couplings .....	169
8.8	An Example: Electron–Phonon and Electron–Plasmon Couplings in $Na_9^+$ .....	171
8.9	Appendices .....	173
8.9.1	Terms Neglected in the Born–Oppenheimer Approximation .....	173
8.9.2	The Electron–Phonon Coupling Constant $\lambda$ .....	175
8.9.3	Outline of the Derivation of (8.50) .....	177
8.9.4	Equation (8.50) Using Second-Order Perturbation Theory .....	178
<b>9.</b>	<b>Pairing in Atomic Aggregates</b> .....	<b>183</b>
9.1	The BCS Solution .....	184
9.2	Superconductivity in Metals .....	190
9.2.1	Finite Temperature .....	190
9.3	Superconductivity in Fullerides .....	194
9.3.1	$C_{60}$ Fullerene Based Superconductors .....	195
9.3.2	Nanometer Superconductors Based on Small Fullerenes .....	197
9.3.3	Thin Carbon Nanotubes .....	200
9.4	Selected Open Questions .....	201
9.4.1	Hole Doped Fullerites .....	202
9.4.2	Screening Effects .....	204
9.5	Appendices .....	204
9.5.1	Simple Estimate of $\mu$ .....	204
9.5.2	Solution of the Pairing Hamiltonian .....	206
9.5.3	Temperature Dependent BCS Solution .....	213
<b>10.</b>	<b>Discussion</b> .....	<b>221</b>
	<b>References</b> .....	<b>224</b>
	<b>Index</b> .....	<b>233</b>