Table of Contents

Preface VII

Introduction 1

1. KdV-Class Solitons 17
1.1 Korteweg–de Vries Equation and KdV-Class Equations 17
1.1.1 Derivation of the KdV Equation 17
1.1.2 Universality of the KdV Model. Scaling Transformations and Similarity Principle 21
1.1.3 Other (1+1)-Dimensional KdV-Class Equations 24
1.2 Inverse Scattering Transform and Analytical Integration 26
1.2.1 Fundamentals of the Inverse Scattering Theory 26
1.2.2 Integration of the KdV Equation Using the IST Method 33
1.2.3 Generalization of the GLM Equation 35
1.2.4 The Variational Principle 40
1.3 Numerical Integration of (1+1)-Dimensional KdV-Class Equations 41
1.3.1 Explicit Difference Schemes 42
1.3.2 Implicit Difference Schemes 43
1.3.3 Remarks on Numerical Integration 48
1.3.4 Test of Numerical Methods, Their Comparative Characteristics, and Use 49
1.3.5 Numerical Solutions of Some KdV-Class Equations ... 50
1.4 Ion-Acoustic Waves in Plasmas 58
1.4.1 The Ion-Acoustic Waves 58
1.4.2 Nonrelativistic Approximation 59
1.4.3 Weakly-Relativistic Effects 61

2. Generalized KdV Equations. NLS and DNLS Equations .. 63
2.1 Generalized KdV Equations 63
2.1.1 The KdV–Burgers Equation. Some Applications 63
2.1.2 Higher Order Dispersion Corrections 67
2.1.3 Modified KdV Equations 69
2.1.4 Higher Order Dispersive Nonlinearity 73

2.2 Structure and Evolution of Solutions of Generalized KdV Equations ... 80
2.2.1 Evolution of Solitons of Generalized KdV Equations ... 80
2.2.2 Soliton Evolution in Media with Stochastic Fluctuations of the Wave Field ... 87
2.2.3 Qualitative Analysis and Asymptotics of Solutions of Generalized KdV-Class Equations ... 92
2.3 Nonlinear Schrödinger and Zakharov Equations ... 104
2.3.1 Derivation of the NLS equation ... 104
2.3.2 IST for the NLS Equation. NLS Solitons ... 107
2.3.3 Zakharov System of Equations ... 111
2.3.4 Langmuir Solitons ... 113
2.3.5 Near-Sonic Solitons ... 116
2.4 Derivative Nonlinear Schrödinger Equation ... 120
2.4.1 Origin of the DNLS Equation ... 120
2.4.2 DNLS Equation as an Integrability Condition for Two Linear Differential Equations ... 123
2.4.3 Stability of DNLS Solitons ... 125
2.4.4 Numerical Approaches to Study Dynamics of Alfvén Solitons ... 127
2.4.5 Results of Numerical Simulations ... 134

3. Classic Two- and Three-Dimensional KP Models and Their Applications ... 137
3.1 (1+2)- and (1+3)-Dimensional KP Equation ... 137
3.1.1 Generalization of the KdV Equation on Weakly Non-One-Dimensional Case ... 137
3.1.2 The KP Equation and its Solutions ... 140
3.1.3 Stability of Two- and Three-Dimensional KP Solitons ... 146
3.1.4 Numerical Approaches to Integration ... 147
3.2 KP Equation: Analytical Integration and Dynamics of Waves 150
3.2.1 Analytical Integration. "Dressing" Method ... 150
3.2.2 Three-Dimensional Inverse Scattering Problem ... 158
3.2.3 Dynamics of Three-Dimensional Nonlinear Waves in the KP Model. Wave Collapse and Self-Focusing ... 165

4. Generalized Two- and Three-Dimensional Models and Their Applications ... 175
4.1 Basic Dynamic Equations in the Long-Wavelength Approximation, Their Generalizations, and Solutions ... 175
4.1.1 Generalized KP Equation ... 175
4.1.2 3-DNLS Equation ... 177
4.1.3 Stability of Two-Dimensional and Three-Dimensional Solutions of GKP and 3-DNLS Equations ... 179

4.2 Asymptotic and Qualitative Analysis of Solutions of GKP Equation and 3-DNLS Equation . . . 187
4.2.1 Basic Equations . . . 187
4.2.2 Generalization to Multidimensional Cases . . . 189
4.2.3 Concluding Remarks . . . 193
4.3 Approaches to Numerical Integration of Equations of GKP-Class and 3-DNLS-Class . . . 194
4.3.1 Groups of Explicit and Implicit Difference Schemes . . . 195
4.3.2 Boundary Conditions and Diffraction Terms . . . 202
4.3.3 Dynamic Spectral Method . . . 204
4.3.4 Comparative Characteristics of Different Schemes and Their Use in Numerical Simulation . . . 208
4.4 Dynamics of Two-Dimensional Solitons in Dispersive Media . . 212
4.4.1 Structure of Two-Dimensional Solutions of GKP-Class Equations . . . 212
4.4.2 Interactions of Two-Dimensional Solitons . . . 218
4.4.3 Influence of Dissipation on Evolution of Two-Dimensional Solitons . . . 222
4.4.4 Evolution of Two-Dimensional Solitons in Media with Stochastic Fluctuations of the Wave Field . . . 223
4.4.5 Structure and Evolution of Two-Dimensional Solitons in Media with Variable Dispersion . . . 229
4.5 Evolution of Three-Dimensional Nonlinear Waves in Dispersive Media . . . 234
4.5.1 Structure and Evolution of Three-Dimensional Solutions of GKP-Class Equations . . . 234
4.5.2 Structure and Evolution of Three-Dimensional Solutions of 3-DNLS-Class Equations . . . 242
4.5.3 Influence of Dissipation on Evolution of Three-Dimensional Nonlinear Waves . . . 246
4.6 Applications . . . 249
4.6.1 Nonlinear Ion-Acoustic Waves in a Plasma . . . 250
4.6.2 Nonlinear Effects in Propagation of FMS Waves in a Magnetized Plasma . . . 255
4.6.3 Solitary Internal Gravity Waves in the F-layer of Earth's Ionosphere . . . 263
4.6.4 Two-Dimensional Solitons in Shallow Water . . . 271

5. Appendices . . . 275

References . . . 279