

# Contents

Foreword	vii
<b>A Saturated and Conditional Structures in Banach Spaces</b> <i>Spiros A. Argyros</i>	<b>1</b>
<b>Introduction</b>	<b>3</b>
<b>I. Tsirelson and Mixed Tsirelson Spaces</b>	<b>7</b>
<b>II. Tree Complete Extensions of a Ground Norm</b>	<b>21</b>
II.1 Mixed Tsirelson Extension of a Ground Norm . . . . .	21
II.2 R.I.S. Sequences and the Basic Inequality . . . . .	26
<b>III. Hereditarily Indecomposable Extensions with a Schauder Basis</b>	<b>39</b>
III.1 The HI Property in $\mathfrak{X}[G, \sigma]$ . . . . .	39
III.2 The HI Property in $\mathfrak{X}[G, \sigma]_*$ . . . . .	43
<b>IV. The Space of the Operators for HI Banach Spaces</b>	<b>47</b>
IV.1 Some General Properties of HI Spaces . . . . .	47
IV.2 The Space of Operators $\mathcal{L}(\mathfrak{X}[G, \sigma])$ , $\mathcal{L}(\mathfrak{X}[G, \sigma]_*)$ . . . . .	52
<b>V. Examples of Hereditarily Indecomposable Extensions</b>	<b>57</b>
V.1 A Quasi-reflexive HI Space . . . . .	57
V.2 The Spaces $\ell_p$ , $1 < p < \infty$ , are Quotients of HI Spaces . . . . .	58
V.3 A Non Separable HI Space . . . . .	62
<b>VI. The Space <math>\mathfrak{X}_{\omega_1}</math></b>	<b>71</b>
<b>VII. Finite Representability of <math>J_{T_0}</math> and the Diagonal Space <math>\mathcal{D}(\mathfrak{X}_\gamma)</math></b>	<b>81</b>
<b>VIII. The Spaces of Operators <math>\mathcal{L}(\mathfrak{X}_\gamma)</math>, <math>\mathcal{L}(X, \mathfrak{X}_{\omega_1})</math></b>	<b>87</b>
<b>Appendix A. Transfinite Schauder Basic Sequences</b>	<b>99</b>
<b>Appendix B. The Proof of the Finite Representability of <math>J_{T_0}</math></b>	<b>105</b>
<b>Bibliography</b>	<b>117</b>

<b>B High-Dimensional Ramsey Theory and Banach Space Geometry</b>	<b>121</b>
<i>Stevo Todorćević</i>	
<b>Introduction</b>	<b>123</b>
<b>I. Finite-Dimensional Ramsey Theory</b>	<b>127</b>
I.1 Finite-Dimensional Ramsey Theorem . . . . .	127
I.2 Spreading Models of Banach Spaces . . . . .	130
I.3 Finite Representability of Banach Spaces . . . . .	135
<b>II. Ramsey Theory of Finite and Infinite Sequences</b>	<b>143</b>
II.1 The Theory of Well-Quasi-Ordered Sets . . . . .	143
II.2 Nash–Williams’ Theory of Fronts and Barriers . . . . .	147
II.3 Uniform Fronts and Barriers . . . . .	153
II.4 Canonical Equivalence Relations on Uniform Fronts and Barriers . . . . .	165
II.5 Unconditional Subsequences of Weakly Null Sequences . . . . .	169
II.6 Topological Ramsey Theory . . . . .	177
II.7 The Theory of Better-Quasi-Orderings . . . . .	180
II.8 Ellentuck’s Theorem . . . . .	185
II.9 Summability in Banach Spaces . . . . .	188
II.10 Summability in Topological Abelian Groups . . . . .	192
<b>III. Ramsey Theory of Finite and Infinite Block Sequences</b>	<b>197</b>
III.1 Hindman’s Theorem . . . . .	197
III.2 Canonical Equivalence Relations on FIN . . . . .	200
III.3 Fronts and Barriers on $\text{FIN}^{< \infty}$ . . . . .	201
III.4 Milliken’s Theorem . . . . .	205
III.5 An Approximate Ramsey Theorem . . . . .	209
<b>IV. Approximate and Strategic Ramsey Theory of Banach Spaces</b>	<b>217</b>
IV.1 Gowers’ Dichotomy . . . . .	217
IV.2 Approximate and Strategic Ramsey Sets . . . . .	220
IV.3 Combinatorial Forcing on Block Sequences in Banach Spaces . . . . .	224
IV.4 Coding into Approximate and Strategic Ramsey Sets . . . . .	229
IV.5 Topological Ramsey Theory of Block Sequences in Banach Spaces . . . . .	233
IV.6 An Application to Rough Classification of Banach Spaces . . . . .	240
IV.7 An Analytic Set whose Complement is not Approximately Ramsey . . . . .	243
<b>Bibliography</b>	<b>247</b>
<b>Index</b>	<b>253</b>