Preface

In the past decades the subject of the spontaneous formation of structures in systems far from equilibrium has grown into a major branch of physics research with strong ties to neighboring disciplines. It has become evident that a diverse range of phenomena can be understood within a common mathematical framework which has been called nonlinear dynamics of continuous systems. This name emphasizes the close relationship to the field of nonlinear dynamics of systems with few degrees of freedom which has evolved into a mathematically mature subject in the recent past. Many dynamical features of continuous systems can actually be described through a reduction to a few degrees of freedom and properties of the latter type of systems continue to inspire the study of continuous systems.

The goal of this book is to demonstrate through numerous examples the opportunities that exist for the study of nonlinear phenomena through the use of common tools of mathematical analyses and dynamical interpretations. Instead of providing a comprehensive overview of the rapidly evolving field, the contributors to this book are trying to communicate to a wide scientific audience the essence of what they have learnt about the formation of spontaneous structures in dissipative continuous systems and about the competition between order and chaos that characterizes these systems. It is hoped that the book will be helpful even to those scientists whose disciplines are not represented by the contributing authors.

The first chapter introduces the mathematical foundations of the subject and some of the mathematical methods used in its study. This chapter clearly plays a central role even though it is not a prerequisite for understanding the following chapters. All chapters of the book are self-contained. But the first one challenges most strongly the mathematical abilities of the reader.

Fluid dynamics traditionally has played a leading role in the development of the nonlinear dynamics of continuous systems. The basic equations of motion are well known in this case and the mathematical analysis has proceeded further into regimes of spatio-temporal chaos than has been possible in other continuous systems. Even the mathematical analysis of the dynamics of complex fluids such as liquid crystals has advanced sufficiently such that quantitative comparisons with observations can be achieved. The seventh and eighth chapters focus on recent progress in this area.

The examples of structure formation in simple Newtonian fluids are far too numerous to be surveyed in the present volume. Several books have been devoted to the classical cases of spontaneous formation of patterns such as Rayleigh–Bénard convection in a fluid layer heated from below and the Taylor–Couette system. Recent progress in the understanding of unusual dynamical features of the latter system is described in the second chapter. Convection driven by centrifugal buoyancy and surface tension induced instabilities are other topics of this chapter. In the third chapter a modified Rayleigh–Bénard problem is discussed where the pure fluid is replaced by a mixture of two fluids. Also addressed in this chapter is the influence of a mean shear flow on problems of pattern formation. Examples of purely hydrodynamic instabilities in shear layers and the subsequent transition to turbulence are considered in the fourth chapter. In comparison with the closed systems treated in most of the other chapters, the formation of patterns cannot be exhibited as easily in open systems and sophisticated methods of data analysis are often required to identify "coherent structures".

In contrast to the usually assumed time independence of the external conditions, the Faraday experiment depends on time periodic variations of the effective gravity acting on a fluid layer with a free surface. A fascinating variety of standing surface waves is observed in this case. Recent theoretical developments together with experimental observations are reported in the fifth chapter. Periodic conditions can also be imposed in the spatial domain. The resulting dynamics gains complexity through the interaction of modes with forced and spontaneous structures. These and other heterogeneity effects are discussed in the sixth chapter. But examples can also be found in other chapters as, for example, in the eighth chapter where the influence of noise on the onset of electrohydrodynamic instability in liquid crystals has been analyzed.

After hydrodynamical systems, chemical oscillations are the next best understood pattern forming systems. Although the details of the chemical reactions are often too complex to be modelled accurately, quantitative comparisons with measurements are still possible on the basis of models with reaction–diffusion equations for just the major chemical species. Particularly interesting examples of pattern dynamics have been found in recent years in chemical reactions on catalytic surfaces, a topic which is presented in the ninth chapter. Because of buoyancy driven flows caused by concentration gradients, chemical reactions in the fluid phase are intimately connected with convection phenomena as is discussed in the tenth chapter. The wide area of pattern formation in flames could be added here if sufficient space were available.

It is remarkable to what extent the reaction–diffusion type equations can be applied for the description of structure forming processes in semiconductors. Especially useful are coupled activator and inhibitor equations. The activator part corresponds to ionisation or to electron–hole pair creation, while the buildup of space charges acts as an inhibitor. In the eleventh chapter a wide variety of experimentally observed phenomena are described within this framework. The basic coherent structure is the current filament. It appears as a single entity or in regular patterns and its dynamics can be influenced by magnetic fields or with electron or laser beams. The numerous electronic techniques available will undoubtedly ensure increasing applications of the formations of structures in semiconductors.

The last three chapters are devoted to subjects that are further removed from the central theme of the book. Granular media such as ordinary sand exhibit a variety of different dynamical patterns when shaken or rotated. But a quantitative continuum description is not yet possible in spite of the similarity between sandripples and wave phenomena. The theoretical interpretations in the chapter on granular materials thus rely primarily on computational simulations of the molecular dynamics type which have become a highly successful tool in studying the dynamics of collections of little solid spheres. The dynamo problem outlined in the next to last chapter, on the other hand, represents a typical bifurcation problem in the dynamics of electrically conducting fluids. There exists little opportunity for pattern formation, however, since the structure of the generated magnetic field will be governed by the outer boundary conditions. This property is caused by the huge magnetic diffusivity of the order of $1 m^2/s$ of typical liquid metals. The formation of spontaneous magnetic structures thus becomes a problem of cosmic dimensions and only with special efforts it may be realized in the laboratory.

Finally, the last chapter presents an example of the formation of spatio-temporal structures in biology and demonstrates the extent to which the development of multicellular organisms can be understood on the basis of reaction–diffusion type equations together with equations of motion. It is fascinating to see how the morphogenetic mechanisms in the evolution of a slime mould can be explained on the basis of rather simple physical and chemical principles.

Acknowledgement: The authors of the chapters of this volume are grateful to the Deutsche Forschungsgemeinschaft for its support of the research focus on Evolution of Spontaneous Structures in Dissipative Continuous Systems. They also wish to thank the evaluation committee and especially its chairman, Prof. S. Großmann, for the guidance it has provided throughout the past seven years.

Bayreuth and Magdeburg, October 1998

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