

# Contents

<b>1. Introduction to scheduling</b> .....	5
1.1 Definition .....	5
1.2 Some areas of application .....	6
1.2.1 Problems related to production .....	6
1.2.2 Other problems .....	7
1.3 Shop environments .....	7
1.3.1 Scheduling problems without assignment .....	8
1.3.2 Scheduling and assignment problems with stages .....	8
1.3.3 General scheduling and assignment problems .....	9
1.4 Constraints .....	9
1.5 Optimality criteria .....	12
1.5.1 Minimisation of a maximum function: “minimax” criteria .....	13
1.5.2 Minimisation of a sum function: “minisum” criteria .....	13
1.6 Typologies and notation of problems .....	14
1.6.1 Typologies of problems .....	14
1.6.2 Notation of problems .....	16
1.7 Project scheduling problems .....	17
1.8 Some fundamental notions .....	18
1.9 Basic scheduling algorithms .....	21
1.9.1 Scheduling rules .....	21
1.9.2 Some classical scheduling algorithms .....	22
<b>2. Complexity of problems and algorithms</b> .....	29
2.1 Complexity of algorithms .....	29
2.2 Complexity of problems .....	32
2.2.1 The complexity of decision problems .....	33
2.2.2 The complexity of optimisation problems .....	38
2.2.3 The complexity of counting and enumeration problems .....	40
2.3 Application to scheduling .....	48
<b>3. Multicriteria optimisation theory</b> .....	53
3.1 MCDA and MCDM: the context .....	53
3.1.1 MultiCriteria Decision Making .....	54

3.1.2	MultiCriteria Decision Aid .....	54
3.2	Presentation of multicriteria optimisation theory .....	55
3.3	Definition of optimality .....	57
3.4	Geometric interpretation using dominance cones .....	60
3.5	Classes of resolution methods .....	62
3.6	Determination of Pareto optima .....	64
3.6.1	Determination by convex combination of criteria .....	64
3.6.2	Determination by parametric analysis .....	70
3.6.3	Determination by means of the $\epsilon$ -constraint approach ..	72
3.6.4	Use of the Tchebycheff metric .....	76
3.6.5	Use of the weighted Tchebycheff metric .....	79
3.6.6	Use of the augmented weighted Tchebycheff metric ...	81
3.6.7	Determination by the goal-attainment approach .....	86
3.6.8	Other methods for determining Pareto optima .....	91
3.7	Multicriteria Linear Programming (MLP) .....	92
3.7.1	Initial results .....	93
3.7.2	Application of the previous results .....	93
3.8	Multicriteria Mixed Integer Programming (MMIP) .....	94
3.8.1	Initial results .....	94
3.8.2	Application of the previous results .....	95
3.8.3	Some classical algorithms .....	97
3.9	The complexity of multicriteria problems .....	100
3.9.1	Complexity results related to the solutions .....	100
3.9.2	Complexity results related to objective functions .....	101
3.9.3	Summary .....	106
3.10	Interactive methods .....	107
3.11	Goal programming .....	108
3.11.1	Archimedian goal programming .....	110
3.11.2	Lexicographical goal programming .....	111
3.11.3	Interactive goal programming .....	111
3.11.4	Reference goal programming .....	112
3.11.5	Multicriteria goal programming .....	112
<b>4.</b>	<b>An approach to multicriteria scheduling problems .....</b>	<b>113</b>
4.1	Justification of the study .....	113
4.1.1	Motivations .....	113
4.1.2	Some examples .....	114
4.2	Presentation of the approach .....	118
4.2.1	Definitions .....	118
4.2.2	Notation of multicriteria scheduling problems .....	121
4.3	Classes of resolution methods .....	122
4.4	Application of the process - an example .....	123
4.5	Some complexity results for multicriteria scheduling problems	124

<b>5. Just-in-Time scheduling problems</b> .....	135
5.1 Presentation of Just-in-Time (JiT) scheduling problems .....	135
5.2 Typology of JiT scheduling problems .....	136
5.2.1 Definition of the due dates .....	136
5.2.2 Definition of the JiT criteria .....	137
5.3 A new approach for JiT scheduling .....	139
5.3.1 Modelling of production costs in JiT scheduling for shop problems .....	141
5.3.2 Links with objective functions of classic JiT scheduling .....	145
5.4 Optimal timing problems .....	147
5.4.1 The $1 d_i, seq F_\ell(\bar{T}^\alpha, \bar{E}^\beta)$ problem .....	147
5.4.2 The $P\infty prec, f_i \text{ convex} \sum_i f_i$ problem .....	149
5.4.3 The $1 f_i \text{ piecewise linear} F_\ell(\sum_i f_i, \sum_j I_j)$ problem .....	153
5.5 Polynomially solvable problems .....	153
5.5.1 The $1 d_i = d \geq \sum p_i F_\ell(\bar{E}, \bar{T})$ problem .....	153
5.5.2 The $1 d_i = d \text{ unknown}, nmit F_\ell(\bar{E}, \bar{T}, d)$ problem .....	155
5.5.3 The $1 p_i \subseteq [\underline{p}_i; \bar{p}_i] \cap \mathbb{N}, d_i = d \text{ non restrictive} F_\ell(\bar{E}, \bar{T}, \overline{CC}^w)$ problem .....	157
5.5.4 The $P d_i = d \text{ non restrictive}, nmit F_\ell(\bar{E}, \bar{T})$ problem .....	157
5.5.5 The $P d_i = d \text{ unknown}, nmit F_\ell(\bar{E}, \bar{T})$ problem .....	159
5.5.6 The $P d_i = d \text{ unknown}, p_i = p, nmit F_\ell(\bar{E}, \bar{T}, d)$ problem .....	165
5.5.7 The $R p_{i,j} \in [\underline{p}_{i,j}; \bar{p}_{i,j}], d_i = d \text{ unknown} F_\ell(\bar{T}, \bar{E}, \overline{CC}^w)$ problem .....	169
5.5.8 Other problems .....	170
5.6 $\mathcal{NP}$ -hard problems .....	173
5.6.1 The $1 d_i, nmit F_\ell(\bar{E}^\alpha, \bar{T}^\beta)$ problem .....	173
5.6.2 The $F prmu, d_i, nmit F_\ell(\bar{E}^w, \bar{T}^w)$ problem .....	176
5.6.3 The $P d_i = d \text{ non restrictive}, nmit f_{max}(\bar{E}^w, \bar{T}^w)$ problem .....	178
5.6.4 Other problems .....	182
5.7 Open problems .....	188
5.7.1 The $Q d_i = d \text{ unknown}, nmit F_\ell(\bar{E}, \bar{T})$ problem .....	188
5.7.2 Other problems .....	189
<b>6. Robustness considerations</b> .....	193
6.1 Introduction to flexibility and robustness in scheduling .....	193
6.2 Approaches that introduce sequential flexibility .....	195
6.2.1 Groups of permutable operations .....	195
6.2.2 Partial order between operations .....	197
6.2.3 Interval structures .....	199
6.3 Single machine problems .....	201
6.3.1 Stability <i>vs</i> makespan .....	201
6.3.2 Robust evaluation <i>vs</i> distance to a baseline solution .....	202

6.4	Flowshop and jobshop problems . . . . .	203
6.4.1	Average makespan of a neighbourhood . . . . .	203
6.4.2	Sensitivity of operations <i>vs</i> makespan . . . . .	203
6.5	Resource Constrained Project Scheduling Problems (RCPSP) . . . . .	204
6.5.1	Quality in project scheduling <i>vs</i> makespan . . . . .	204
6.5.2	Stability <i>vs</i> makespan . . . . .	205
<b>7.</b>	<b>Single machine problems</b> . . . . .	<b>207</b>
7.1	Polynomially solvable problems . . . . .	207
7.1.1	Some $1 d_i \overline{C}, f_{\max}$ problems . . . . .	207
7.1.2	The $1 s_i, pmtn, nmit F_{\ell}(\overline{C}, P_{\max})$ problem . . . . .	215
7.1.3	The $1 p_i \in [\underline{p}_i; \overline{p}_i], d_i F_{\ell}(T_{max}, \overline{CC}^w)$ problem . . . . .	216
7.1.4	The $1 p_i \in [\underline{p}_i; \overline{p}_i], d_i F_{\ell}(\overline{C}, \overline{CC}^w)$ problem . . . . .	219
7.1.5	Other problems . . . . .	219
7.2	$\mathcal{NP}$ -hard problems . . . . .	222
7.2.1	The $1 d_i \overline{T}, \overline{C}$ problem . . . . .	222
7.2.2	The $1 r_i, p_i \in [\underline{p}_i; \overline{p}_i] \cap \mathbb{N} F_{\ell}(C_{max}, \overline{CC}^w)$ problem . . . . .	223
7.2.3	The $1 r_i, p_i \in [\underline{p}_i; \overline{p}_i] \cap \mathbb{N} F_{\ell}(\overline{U}^w, \overline{CC}^w)$ problem . . . . .	225
7.2.4	Other problems . . . . .	226
7.3	Open problems . . . . .	230
7.3.1	The $1 d_i \overline{U}, T_{max}$ problem . . . . .	230
7.3.2	Other problems . . . . .	234
<b>8.</b>	<b>Shop problems</b> . . . . .	<b>235</b>
8.1	Two-machine flowshop problems . . . . .	235
8.1.1	The $F2 prmu Lex(C_{max}, \overline{C})$ problem . . . . .	235
8.1.2	The $F2 prmu F_{\ell}(C_{max}, \overline{C})$ problem . . . . .	250
8.1.3	The $F2 prmu, r_i F_{\ell}(C_{max}, \overline{C})$ problem . . . . .	256
8.1.4	The $F2 prmu \epsilon(\overline{C}/C_{max})$ problem . . . . .	256
8.1.5	The $F2 prmu, d_i \#(C_{max}, T_{max})$ problem . . . . .	262
8.1.6	The $F2 prmu, d_i \#(C_{max}, \overline{U})$ problem . . . . .	265
8.1.7	The $F2 prmu, d_i \#(C_{max}, \overline{T})$ problem . . . . .	267
8.2	$m$ -machine flowshop problems . . . . .	270
8.2.1	The $F prmu Lex(C_{max}, \overline{C})$ problem . . . . .	270
8.2.2	The $F prmu \#(C_{max}, \overline{C})$ problem . . . . .	272
8.2.3	The $F prmu, d_i \epsilon(C_{max}/T_{max})$ problem . . . . .	277
8.2.4	The $F p_{i,j} \in [\underline{p}_{i,j}; \overline{p}_{i,j}], prmu F_{\ell}(C_{max}, \overline{CC}^w)$ problem . . . . .	280
8.2.5	The $F p_{i,j} = p_i \in [\underline{p}_i; \overline{p}_i], prmu \#(C_{max}, \overline{CC}^w)$ problem . . . . .	281
8.3	Jobshop and Openshop problems . . . . .	284
8.3.1	Jobshop problems . . . . .	284
8.3.2	The $O2  Lex(C_{max}, \overline{C})$ problem . . . . .	284
8.3.3	The $O3  Lex(C_{max}, \overline{C})$ problem . . . . .	286

<b>9. Parallel machines problems</b> .....	287
9.1 Problems with identical parallel machines .....	287
9.1.1 The $P2 pmtn, d_i \epsilon(L_{max}/C_{max})$ problem .....	287
9.1.2 The $P3 pmtn, d_i \epsilon(L_{max}/C_{max})$ problem .....	290
9.1.3 The $P2 d_i Lex(T_{max}, \bar{U})$ problem .....	293
9.1.4 The $P d_i \#(\bar{C}, \bar{U})$ problem .....	295
9.1.5 The $P pmtn Lex(\bar{C}, C_{max})$ problem .....	296
9.2 Problems with uniform parallel machines .....	297
9.2.1 The $Q p_i = p \epsilon(f_{max}/g_{max})$ problem .....	297
9.2.2 The $Q p_i = p \epsilon(\bar{g}/f_{max})$ problem .....	302
9.2.3 The $Q pmtn \epsilon(\bar{C}/C_{max})$ problem .....	303
9.3 Problems with unrelated parallel machines .....	310
9.3.1 The $R p_{i,j} \in [p_{i,j}, \bar{p}_{i,j}] F_\ell(\bar{C}, \bar{C}^w)$ problem .....	310
9.3.2 The $R pmtn \epsilon(F_\ell(I_{max}, \bar{M})/C_{max})$ problem .....	310
<b>10. Shop problems with assignment</b> .....	315
10.1 A hybrid flowshop problem with three stages .....	315
10.2 Hybrid flowshop problems with $k$ stages .....	316
10.2.1 The $HFk, (PM^{(\ell)})_{\ell=1}^k    F_\ell(C_{max}, \bar{C})$ problem .....	316
10.2.2 The $HFk, (PM^{(\ell)})_{\ell=1}^k    \epsilon(\bar{C}/C_{max})$ problem .....	318
10.2.3 The $HFk, (PM^{(\ell)}(t))_{\ell=1}^k  r_i^{(1)}, d_i^{(k)} \epsilon(C_{max}/T_{max})$ problem .....	318
<b>A. Notations</b> .....	323
A.1 Notation of data and variables .....	323
A.2 Usual notation of single criterion scheduling problems .....	323
<b>B. Synthesis on multicriteria scheduling problems</b> .....	329
B.1 Single machine Just-in-Time scheduling problems .....	329
B.2 Single machine problems .....	330
B.3 Shop problems .....	333
B.4 Parallel machines scheduling problems .....	333
B.5 Shop scheduling problems with assignment .....	334
<b>References</b> .....	335
<b>Index</b> .....	357