Contents

Introduction 1

1 Optimal Periodic Control. The Integral Equations Method . . 5

1.1 Linear Systems, Basic Definitions 6
1.1.1 General Concepts and Definitions. 6
1.1.2 Transfer Function of Linear System. Stable and Physically Realizable Systems 8
1.1.3 Steady and Periodic Solutions of a Linear System 10
1.1.4 Dynamic Characteristics of Mechanical System. 12
1.2 Periodic Green's Functions and Periodic Motions of Linear Systems 15
1.2.1 Periodic Regime of Linear Systems 15
1.2.2 Main Properties of the Periodic Green Function 17
1.2.3 System's Response to the Excitation with Half–Period Sign Change. Periodic Green Function of the Second Kind . . . 20
1.2.4 Integral Equations of Periodic Oscillations of Nonlinear Systems 23
1.3 Necessary Conditions of Optimality for Periodic Regimes 24
1.4 Optimal Periodic Control for Linear Systems (Non-Resonant Case) 31
1.5 Problems of Optimal Displacement for Linear Systems 41
1.5.1 Systems with Symmetric Limiters 42
1.5.2 Systems with Asymmetric Characteristics 46
1.5.3 Systems with Asymmetric Limiters 50
1.6 Periodic Control for Quasi-Linear Systems 52
1.6.1 Periodic Control in Systems Described by Differential Equations 52
1.6.2 The Method of Successive Approximations for Integral Equations of Periodic Movement 54
1.6.3 The Method of Successive Approximations in Problems of the Optimal High-Speed Action 61

2 Periodic Control for Vibroimpact Systems 65

2.1 Motion Equations of Vibroimpact Systems. Integral Equations of

Periodic Motions 67
2.2 Resonant and Quasi-Resonant Oscillations of Vibroimpact Systems 75
2.2.1 Oscillations of Conservative Systems with One Degree of Freedom 75
2.2.2 Resonant Oscillations of Systems with Several Degrees of Freedom 81
2.3 Optimal Periodic Control for Vibroimpact System, Linear between Impacts 82
2.3.1 Control for the Fixed Oscillation Period 82
2.3.2 The Choice of Optimal Period between Impacts 86
2.3.3 Determination of Optimal Clearance (Press Fit). 87
2.3.4 Optimal Control in Systems with Double-sided Symmetric Limiters 89
2.4 Optimal Control for Quasi-Resonant Systems 92
2.4.1 General Equations of the Method of Successive Approximations for Search of Periodic Solutions for Vibroimpact Systems . . 93
2.4.2 Optimal Control of Quasi-Resonant Motions of Vibroimpact Systems 97
2.4.3 Choice of Minimal Duration of Working Cycle 103
2.4.4 Control of Non-Autonomous Quasi-Resonant Systems . . 104
2.4.5 Approximate Optimal Control Synthesis for Vibroimpact Systems 106
2.4.6 Control of Asymmetric Vibroimpact Systems. 108

3 The Averaging Method in Oscillation Control Problems 111

3.1 Optimal Control for Finite Time Interval. Problems of the Optimal High–Speed Action 112
3.1.1 Motion Equations for Systems with Weak Control 112
3.1.2 Problem Formulation. General Equations 113
3.2 Periodic Control 124
3.3 Processes of Oscillation Settlement in Vibroimpact Systems 131
3.3.1 General Equations and Replacement of Variables 131
3.3.2 Main Equations of Motion Control 136
3.3.3 Periodic Control of Quasi-Conservative Systems 142
3.3.4 Partial Averaging. 144
3.3.5 Main Motion Equations of the System with Double-Sided Constraints 146

4 Oscillations in Systems with Random Disturbances 149

4.1 Stochastic Differential Equations. 150
4.2 Limit Theorems for Stochastic Differential Equations (The Diffusion Approximation Method) 157

4.3 Stationary Regimes in Systems with Random Disturbances 171
4.3.1 General Definitions 171
4.3.2 Convergence of Disturbed Motion to a Limit Homogenous Diffusion Process 172
4.4 Oscillations of Vibroimpact Systems at Random Disturbance 177

5 Some Problems of Optimal Control for Systems with Random Disturbances 183

5.1 Program Control in Systems with Random Disturbances 184
5.1.1 Necessary Conditions for Optimal Program Control in Stochastic Systems 185
5.1.2 Program Control for Systems with Wide–Band Disturbances 188
5.1.3 Periodic Control of Parametric Disturbances of Linear Systems 191
5.2 The Method of Dynamic Programming for Optimal Control Synthesis for Disturbed Systems 197
5.2.1 Equations of Dynamic Programming 199
5.2.2 Optimal Control for Systems with Wide–Band Random Disturbances [79] 202
5.2.3 Periodic Control for Systems with Disturbances 207
5.3 Control for Stationary Motion under Random Disturbances . . . 213
5.3.1 Stationary Quality Criterion 213
5.3.2 Control for Stationary Motion in Systems with Wide–Band Random Disturbances 217

A Appendix 227

A.1 Pontryagin Maximum Principle 227
A.2 Disturbances in Optimal Systems 230
A.3 Main Theorems of the Averaging Method 232
A.4 Necessary Condition of the Optimality of Periodic Regimes 241
A.5 Maximum Principle for Stochastic Equations with Program Control 242
A.6 Main Theorems of the Diffusion Approximation Method 245

References 257