

Contents

1	The Complexity of Optimization Problems	1
1.1	Analysis of algorithms and complexity of problems	2
1.1.1	Complexity analysis of computer programs	3
1.1.2	Upper and lower bounds on the complexity of problems	8
1.2	Complexity classes of decision problems	9
1.2.1	The class NP	12
1.3	Reducibility among problems	17
1.3.1	Karp and Turing reducibility	17
1.3.2	NP-complete problems	21
1.4	Complexity of optimization problems	22
1.4.1	Optimization problems	22
1.4.2	PO and NPO problems	26
1.4.3	NP-hard optimization problems	29
1.4.4	Optimization problems and evaluation problems .	31
1.5	Exercises	33
1.6	Bibliographical notes	36
2	Design Techniques for Approximation Algorithms	39
2.1	The greedy method	40
2.1.1	Greedy algorithm for the knapsack problem	41
2.1.2	Greedy algorithm for the independent set problem	43
2.1.3	Greedy algorithm for the salesperson problem . . .	47

2.2	Sequential algorithms for partitioning problems	50
2.2.1	Scheduling jobs on identical machines	51
2.2.2	Sequential algorithms for bin packing	53
2.2.3	Sequential algorithms for the graph coloring problem	58
2.3	Local search	61
2.3.1	Local search algorithms for the cut problem	62
2.3.2	Local search algorithms for the salesperson problem	64
2.4	Linear programming based algorithms	65
2.4.1	Rounding the solution of a linear program	66
2.4.2	Primal-dual algorithms	67
2.5	Dynamic programming	69
2.6	Randomized algorithms	74
2.7	Approaches to the approximate solution of problems	76
2.7.1	Performance guarantee: chapters 3 and 4	76
2.7.2	Randomized algorithms: chapter 5	77
2.7.3	Probabilistic analysis: chapter 9	77
2.7.4	Heuristics: chapter 10	78
2.7.5	Final remarks	79
2.8	Exercises	79
2.9	Bibliographical notes	83
3	Approximation Classes	87
3.1	Approximate solutions with guaranteed performance	88
3.1.1	Absolute approximation	88
3.1.2	Relative approximation	90
3.1.3	Approximability and non-approximability of TSP	94
3.1.4	Limits to approximability: The gap technique	100
3.2	Polynomial-time approximation schemes	102
3.2.1	The class PTAS	105
3.2.2	APX versus PTAS	110
3.3	Fully polynomial-time approximation schemes	111
3.3.1	The class FPTAS	111
3.3.2	The variable partitioning technique	112
3.3.3	Negative results for the class FPTAS	113
3.3.4	Strong NP-completeness and pseudo-polynomiality	114
3.4	Exercises	116
3.5	Bibliographical notes	119
4	Input-Dependent and Asymptotic Approximation	123
4.1	Between APX and NPO	124
4.1.1	Approximating the set cover problem	124
4.1.2	Approximating the graph coloring problem	127

4.1.3	Approximating the minimum multi-cut problem . . .	129
4.2	Between APX and PTAS	139
4.2.1	Approximating the edge coloring problem	139
4.2.2	Approximating the bin packing problem	143
4.3	Exercises	148
4.4	Bibliographical notes	150
5	Approximation through Randomization	153
5.1	Randomized algorithms for weighted vertex cover	154
5.2	Randomized algorithms for weighted satisfiability	157
5.2.1	A new randomized approximation algorithm	157
5.2.2	A 4/3-approximation randomized algorithm	160
5.3	Algorithms based on semidefinite programming	162
5.3.1	Improved algorithms for weighted 2-satisfiability	167
5.4	The method of the conditional probabilities	168
5.5	Exercises	171
5.6	Bibliographical notes	173
6	NP, PCP and Non-approximability Results	175
6.1	Formal complexity theory	175
6.1.1	Turing machines	175
6.1.2	Deterministic Turing machines	178
6.1.3	Nondeterministic Turing machines	180
6.1.4	Time and space complexity	181
6.1.5	NP-completeness and Cook-Levin theorem	184
6.2	Oracles	188
6.2.1	Oracle Turing machines	189
6.3	The PCP model	190
6.3.1	Membership proofs	190
6.3.2	Probabilistic Turing machines	191
6.3.3	Verifiers and PCP	193
6.3.4	A different view of NP	194
6.4	Using PCP to prove non-approximability results	195
6.4.1	The maximum satisfiability problem	196
6.4.2	The maximum clique problem	198
6.5	Exercises	200
6.6	Bibliographical notes	204
7	The PCP theorem	207
7.1	Transparent long proofs	208
7.1.1	Linear functions	210
7.1.2	Arithmetization	214

7.1.3	The first PCP result	218
7.2	Almost transparent short proofs	221
7.2.1	Low-degree polynomials	222
7.2.2	Arithmetization (revisited)	231
7.2.3	The second PCP result	238
7.3	The final proof	239
7.3.1	Normal form verifiers	241
7.3.2	The composition lemma	245
7.4	Exercises	248
7.5	Bibliographical notes	249
8	Approximation Preserving Reductions	253
8.1	The World of NPO Problems	254
8.2	AP-reducibility	256
8.2.1	Complete problems	261
8.3	NPO-completeness	261
8.3.1	Other NPO-complete problems	265
8.3.2	Completeness in exp-APX	265
8.4	APX-completeness	266
8.4.1	Other APX-complete problems	270
8.5	Exercises	281
8.6	Bibliographical notes	283
9	Probabilistic analysis of approximation algorithms	287
9.1	Introduction	288
9.1.1	Goals of probabilistic analysis	289
9.2	Techniques for the probabilistic analysis of algorithms	291
9.2.1	Conditioning in the analysis of algorithms	291
9.2.2	The first and the second moment methods	293
9.2.3	Convergence of random variables	294
9.3	Probabilistic analysis and multiprocessor scheduling	296
9.4	Probabilistic analysis and bin packing	298
9.5	Probabilistic analysis and maximum clique	302
9.6	Probabilistic analysis and graph coloring	311
9.7	Probabilistic analysis and Euclidean TSP	312
9.8	Exercises	316
9.9	Bibliographical notes	318
10	Heuristic methods	321
10.1	Types of heuristics	322
10.2	Construction heuristics	325
10.3	Local search heuristics	329

10.3.1	Fixed-depth local search heuristics	330
10.3.2	Variable-depth local search heuristics	336
10.4	Heuristics based on local search	341
10.4.1	Simulated annealing	341
10.4.2	Genetic algorithms	344
10.4.3	Tabu search	347
10.5	Exercises	349
10.6	Bibliographical notes	350
A	Mathematical preliminaries	353
A.1	Sets	353
A.1.1	Sequences, tuples and matrices	354
A.2	Functions and relations	355
A.3	Graphs	356
A.4	Strings and languages	357
A.5	Boolean logic	357
A.6	Probability	358
A.6.1	Random variables	359
A.7	Linear programming	361
A.8	Two famous formulas	365
B	A List of NP Optimization Problems	367
	Bibliography	471
	Index	515