

# CONTENTS

## LECTURE 1

### Fundamentals of Optical Soliton Theory in Fibers

by A. Hasegawa

1. Introduction .....	1
2. Electromagnetic waves in dielectric materials.....	1
2.1 Polarization effects .....	1
2.2 Plane electromagnetic waves in dielectric materials.....	3
2.3 Kerr effect and Kerr coefficient.....	5
2.4 Dielectric waveguides .....	6
3. Envelope of electromagnetic wave in dielectric materials.....	10
3.1 Lightwave envelope in fibers – derivation of nonlinear Schrödinger equation .....	10
3.2 Evolution of the wave packet due to the group velocity dispersion	12
3.3 Evolution of wave packet due to the nonlinearity .....	14
3.4 Lax theorem.....	15
3.5 The soliton solution of the nonlinear Schrödinger equation .....	15
4. Ultrafast communication based on optical solitons .....	16
5. Conclusion .....	18

## LECTURE 2

### Hamiltonian Theory of Bäcklund Transformation

by V.G. Marikhin and A.B. Shabat

1. Introduction .....	19
2. Lattice equations .....	22
3. Canonical Bäcklund transformations .....	25
4. First integrals .....	27

## LECTURE 3

### Stability of Solitons

by E.A. Kuznetsov

1. Introduction .....	31
2. Lyapunov stability .....	33
2.1 Nonlinear Schrödinger equation .....	33
2.2 The three-wave system .....	36
2.3 Soliton solutions of the 3-wave system .....	39
2.4 Nonlinear stability .....	41
3. Linear stability .....	43
3.1 Linear stability for 1D NLS solitons.....	43
3.2 Solitons for the FF-SH interaction .....	46

## LECTURE 4

### Chaotic Dynamics of Optical Solitons

by F.Kh. Abdullaev

1. Introduction .....	51
2. Variational approach to solitons dynamics in random media .....	52
2.1 Optical solitons in media with fluctuating quadratic potential .....	53
2.2 Spatial soliton in array with fluctuating parameters .....	54
2.3 A random Kepler problem .....	55
3. Inverse scattering transform technique for solitons in random media ....	56
3.1 Single soliton propagation in random media.....	56
3.2 Interaction of optical solitons in random media.....	59
4. Conclusion .....	61

## LECTURE 5

### Variationalism and Empirio-Criticism.

#### (Exact and Variational Approaches to Fibre Optics Equations)

by A.V. Mikhailov

1. Introduction .....	63
2. Variational approach .....	64
3. What is wrong with the Variational approach .....	68

**LECTURE 6****Propagation of Optical Pulses in Nonlinear Systems  
with Varying Dispersion**

by V.E. Zakharov

1. Introduction .....	73
2. Basic model .....	74
3. Effective Hamiltonian .....	77
4. Monochromatic wave and its stability .....	81
5. Weak dispersion management .....	82
6. Strong dispersion management (SDM) .....	85
7. Solitons and their stability .....	87

**LECTURE 7****Dispersion-Managed Solitons**by S.K. Turitsyn, N.J. Doran, J.H.B. Nijhof, V.K. Mezentsev,  
T. Schäfer and W. Forysiak

1. Introduction .....	91
2. Basic equations .....	94
3. Linear solution and qualitative description of DM solitons .....	96
4. DM pulse evolution over one period .....	98
4.1 Root-mean-square momentum equations .....	98
4.2 Power enhancement .....	105
4.3 How to find the DM soliton numerically .....	106
5. A path-average theory of DM solitons in the time domain .....	106
6. Path-averaged equations in the spectral domain .....	110
7. Conclusions .....	112

**LECTURE 8****Dispersion-Managed Solitons: Applications to Terabit/s  
Transmission over Transoceanic Distances**

by T. Georges

Introduction .....	117
Modelling .....	119

Single pulse propagation.....	119
Perturbation .....	125
Experiments.....	135
Set-up .....	135
Spectrum evolution .....	136
Phase diagram .....	136
System margin .....	136
Comparison to a soliton transmission system with constant dispersion .	139
Narrow band 1.02 Tbit/s ( $51 \times 20$ Gbit/s) soliton DWDM transmission over 1000 km of Standard fibre.....	140
Conclusion .....	141

## LECTURE 9

### Nonlinear Pulses in Ultra-Fast Optical Communications

by V. Cautaerts, Y. Kodama, A. Maruta and H. Sugahara

1. Introduction .....	147
2. The DM solitons.....	148
2.1 The Lagrangian method.....	149
2.2 Hermite-Gaussian ansatz .....	151
3. The DM solitons in WDM .....	153
3.1 Mechanism of frequency shift for DM soliton .....	153
3.2 Optimal allocation of amplifier .....	156
3.3 Statistical analysis of collision induced timing jitter .....	158
4. NRZ pulse propagation.....	161
4.1 The NLS-Whitham equations .....	163
4.2 Control of NRZ pulse .....	165

## LECTURE 10

### Soliton Wavelength-Division-Multiplexing System: From Numerical Design to Recirculating Loop Experiments

by J.-P. Hamaide, B. Biotteau, F. Pitel and E. Desurvire

1. Introduction .....	171
2. Soliton transmission over dispersion-managed systems .....	173

3. Results from the analytical/basic numerical tool .....	175
4. Results from the numerical tool .....	177
5. Results from the experimental tool .....	178
6. Conclusion .....	181

## LECTURE 11

### **Mode-Locked Fiber Ring Lasers and Fiber Ring Memories**

by H.A. Haus

1. Introduction .....	183
2. The passively mode-locked fiber ring laser and the master equation .....	184
3. Harmonic modelocking and the makings of an all-optical memory .....	190
4. The first order soliton .....	199
5. Perturbation theory of solitons .....	200
6. The stretched pulse fiber ring laser .....	208

## LECTURE 12

### **Modulational Instabilities in Passive Cavities: Theory and Experiment**

by M. Haelterman and S. Coen

1. Introduction .....	215
2. Basic properties of the nonlinear fiber resonator .....	217
3. The effects of dispersion: Theory .....	222
3.1 cw-MI and the MI-induced up-switching process .....	223
3.2 Period-doubling MI .....	225
4. Experimental results .....	226
4.1 Period-doubling MI .....	227
4.2 cw-MI and the MI-induced up-switching process .....	229
5. Conclusion .....	230

**LECTURE 13****Recent Developments in the Theory  
of Optical Gap Solitons**

by S. Trillo, C. Conti, A. de Rossi and G. Assanto

1. Introduction .....	233
2. Coupled-mode models.....	234
3. Stability.....	236
4. Quadratic gap solitons .....	242
5. Conclusions .....	246

**LECTURE 14****Vector Modulational Instabilities and Soliton Experiments**by G. Millot, S. Pitois, E. Seve, P. Tchofo Dinda, P. Grelu,  
S. Wabnitz, M. Haelterman and S. Trillo

1. Introduction .....	249
2. Observation of vector MI for normal dispersion.....	250
2.1 High-birefringence fiber .....	250
2.2 Low-birefringence fiber .....	251
2.3 Bimodal fiber .....	252
3. MI gain spectra from linear stability analysis .....	252
3.1 High-birefringence fiber .....	253
3.2 Low-birefringence fiber .....	253
3.3 Bimodal fiber .....	254
4. Induced vector MI and soliton generation .....	255
4.1 High-birefringence fiber .....	255
4.2 Low-birefringence fiber .....	258
4.3 Bimodal fiber .....	260
5. Conclusions .....	262

**LECTURE 15****Transient Raman Amplification**

by J. Leon and A.V. Mikhailov

1. Introduction .....	265
-----------------------	-----

2. Derivation of the SRS system .....	269
3. Steady state regime .....	273
4. Transient SRS: A complete solution .....	274
5. Raman soliton generation .....	276
6. Stokes phase flips and the Raman spike .....	277
7. The Raman spike in the time domain.....	278
8. Conclusion .....	280

## LECTURE 16

### Self-Structuration of Three-Wave Dissipative Solitons in CW-Pumped Optical Cavities

by C. Montes, A. Picozzi and M. Haelterman

1. Introduction .....	283
2. Three-wave model .....	284
3. Two-wave adiabatic approximation .....	286
4. Self-pulsing in a cavity .....	289

## LECTURE 17

### The Description of the Ultrashort Pulse Propagation in Non-Linear Media Under Quasi-Resonance Condition

by A.I. Maimitsov

1. Introduction .....	293
2. Maxwell-Bloch, RMB, and SVEPA equations .....	295
3. Solution of the Bloch equation .....	297
4. Scalar wave equations .....	299
4.1 Non-linear wave equation .....	299
4.2 Unidirectional non-linear wave (mKdV equation) .....	302
4.3 Non-linear wave in SVEPA .....	303
5. Vector waves .....	304
5.1 Generalised Maxwell-Bloch equations .....	305
5.2 Solution of the generalised Bloch equations .....	305
5.3 Vector non-linear wave equation .....	307
5.4 Unidirectional vector non-linear waves .....	308

5.5 Polarised quasi-monochromatic non-linear wave (vector NLS equation) .....	309
6. Conclusion .....	310

## LECTURE 18

### Bright Spatial Soliton Interactions

by G.I. Stegeman and M. Segev

1. Introduction .....	313
2. Coherent interactions: Basic theoretical properties .....	316
2.1 Kerr nonlinearities .....	317
2.2 Saturating nonlinearities .....	321
3. Coherent interactions: Experiments .....	322
4. Incoherent soliton interactions .....	324
5. Full 3D soliton interactions .....	326
6. Anisotropic soliton interactions .....	329
7. Summary .....	330

## LECTURE 19

### Spatial Solitons in Saturating Nonlinear Materials

by B. Luther-Davies, V. Tikhonenko, J. Christou,  
W. Krolikowski, Y. Kivshar and N. Akmediev

1. Introduction .....	335
2. Dark and bright spatial solitons .....	338
3. Saturating nonlinearities .....	340
4. Experimental demonstrations .....	341
5. Conclusions .....	346

## LECTURE 20

### Discrete Solitons in Nonlinear Waveguide Arrays

by F. Lederer and J.S. Aitchison

1. Introduction .....	349
2. Basic properties of waveguide arrays .....	352



2.1	Evolution equations .....	352
2.2	Linear properties – “discrete diffraction” .....	353
2.3	Nonlinear properties – modulational instability.....	354
3.	Discrete Solitons .....	355
3.1	Moderately localized bright solitons – basic properties .....	355
3.2	Moderately localized bright solitons – self-trapping and switching .....	358
3.3	Strongly localized discrete solitons – properties and stability .....	359
4.	Further studies.....	360
5.	Experiments in nonlinear waveguide arrays.....	361
6.	Conclusions .....	364

## LECTURE 21

### Solitons in Cavities with Quadratic Nonlinearities

by W.E. Torruellas, P.S. Jian, S. Trillo, M. Haelterman,  
U. Peschel and F. Lederer

Introduction .....	367
1. The case of quadratic nonlinearities .....	368
2. Why cavities?.....	369
3. Multidimensional spatial solitons in optical cavities .....	370
4. Optical bullets in nonlinear optical cavities .....	370
5. Temporal solitons in singly resonant optical parametric oscillators.....	371
6. Conclusion .....	379