Contents

1. Relativistic Wave Equation for Spin-0 Particles: The Klein–Gordon Equation and Its Applications 1

1.1 The Notation 2
1.2 The Klein–Gordon Equation 4
1.3 The Nonrelativistic Limit 7
1.4 Free Spin-0 Particles 8
1.5 Energy-Momentum Tensor of the Klein–Gordon Field 12
1.6 The Klein–Gordon Equation in Schrödinger Form 21
1.7 Charge Conjugation 26
1.8 Free Spin-0 Particles in the Feshbach–Villars Representation 31
1.9 The Interaction of a Spin-0 Particle with an Electromagnetic Field 41
1.10 Gauge Invariance of the Coupling 49
1.11 The Nonrelativistic Limit with Fields 50
1.12 Interpretation of One-Particle Operators in Relativistic Quantum Mechanics 68
1.13 Biographical Notes 97

2. A Wave Equation for Spin-$\frac{1}{2}$ Particles: The Dirac Equation 99

2.1 Free Motion of a Dirac Particle 107
2.2 Single-Particle Interpretation of the Plane (Free) Dirac Waves . . . 111
2.3 Nonrelativistic Limit of the Dirac Equation 120
2.4 Biographical Notes 126

3. Lorentz Covariance of the Dirac Equation 127

3.1 Formulation of Covariance (Form Invariance) 130
3.2 Construction of the $\hat{S}$ Operator for Infinitesimal Lorentz Transformations 140
3.3 Finite Proper Lorentz Transformations 143
3.4 The $\hat{S}$ Operator for Proper Lorentz Transformations 144
3.5 The Four-Current Density 147
3.6 Biographical Notes 148

4. Spinors Under Spatial Reflection 149

5. Bilinear Covariants of the Dirac Spinors 151

5.1 Biographical Notes 156

6. Another Way of Constructing Solutions of the Free Dirac Equation: Construction by Lorentz Transformations 157
6.1 Plane Waves in Arbitrary Directions 161
6.2 The General Form of the Free Solutions and Their Properties . . . 165
6.3 Polarized Electrons in Relativistic Theory 174

7. Projection Operators for Energy and Spin 177
7.1 Simultaneous Projections of Energy and Spin 181

8. Wave Packets of Plane Dirac Waves 183

9. Dirac Particles in External Fields: Examples and Problems 197

10. The Two-Centre Dirac Equation 261

11. The Foldy–Wouthuysen Representation for Free Particles 277
11.1 The Foldy–Wouthuysen Representation in the Presence of External Fields 285

12. The Hole Theory 291
12.1 Charge Conjugation 299
12.2 Charge Conjugation of Eigenstates with Arbitrary Spin and Momentum 309
12.3 Charge Conjugation of Bound States 310
12.4 Time Reversal and PCT Symmetry 312
12.5 Biographical Notes 323

13. Klein's Paradox 325

14. The Weyl Equation – The Neutrino 333

15. Wave Equations for Particles with Arbitrary Spins 347
15.1 Particles with Finite Mass 347
15.2 Massless Particles 355
15.3 Spin-1 Fields for Particles with Finite Mass: Proca Equations . . . 359
15.4 Kemmer Equaton 361
15.5 The Maxwell Equations 364
15.6 Spin-$\frac{3}{2}$ Fields 383
15.7 Biographical Notes 388

16. Lorentz Invariance and Relativistic Symmetry Principles 389
16.1 Orthogonal Transformations in Four Dimensions 389
16.2 Infinitesimal Transformations and the Proper Subgroup of O(4) . 390
16.3 Classification of the Subgroups of O(4) 396
16.4 The Inhomogeneous Lorentz Group 398
16.5 The Conformal Group 400
16.6 Representations of the Four-Dimensional Orthogonal Group and Its Subgroups 402
16.6.1 Tensor Representation of the Proper Groups 402
16.6.2 Spinor Representations 403

16.7 Representation of SL(2, C) 406
16.8 Representations of SO(3, R) 407
16.9 Representations of the Lorentz Group L_p 408
16.10 Spin and the Rotation Group 410
16.11 Biographical Notes 415

Subject Index 417