

# Contents

|   |           |
|---|-----------|
| <b>1. Basic Quantum Mechanics . . . . .</b>                   | <b>1</b>  |
| 1.1 Postulates of Quantum Mechanics . . . . .                 | 1         |
| 1.1.1 Postulate 1 . . . . .                                   | 1         |
| 1.1.2 Postulate 2 . . . . .                                   | 11        |
| 1.1.3 Postulate 3 . . . . .                                   | 11        |
| 1.1.4 Postulate 4 . . . . .                                   | 11        |
| 1.1.5 Postulate 5 . . . . .                                   | 13        |
| 1.2 Geometric Phase . . . . .                                 | 16        |
| 1.2.1 Geometric Phase of a Harmonic Oscillator . . . . .      | 18        |
| 1.2.2 Geometric Phase of a Two-Level System . . . . .         | 18        |
| 1.2.3 Geometric Phase in Adiabatic Evolution . . . . .        | 18        |
| 1.3 Time-Dependent Approximation Method . . . . .             | 19        |
| 1.4 Quantum Mechanics of a Composite System . . . . .         | 20        |
| 1.5 Quantum Mechanics of a Subsystem and Density Operator . . | 21        |
| 1.6 Systems of One and Two Spin-1/2s . . . . .                | 23        |
| 1.7 Wave-Particle Duality . . . . .                           | 26        |
| 1.8 Measurement Postulate and Paradoxes of Quantum Theory .   | 29        |
| 1.8.1 The Measurement Problem . . . . .                       | 30        |
| 1.8.2 Schrödinger's Cat Paradox . . . . .                     | 31        |
| 1.8.3 EPR Paradox . . . . .                                   | 32        |
| 1.9 Local Hidden Variables Theory . . . . .                   | 34        |
| <b>2. Algebra of the Exponential Operator . . . . .</b>       | <b>37</b> |
| 2.1 Parametric Differentiation of the Exponential . . . . .   | 37        |
| 2.2 Exponential of a Finite-Dimensional Operator . . . . .    | 38        |
| 2.3 Lie Algebraic Similarity Transformations . . . . .        | 39        |
| 2.3.1 Harmonic Oscillator Algebra . . . . .                   | 41        |
| 2.3.2 The $SU(2)$ Algebra . . . . .                           | 42        |
| 2.3.3 The $SU(1,1)$ Algebra . . . . .                         | 43        |
| 2.3.4 The $SU(m)$ Algebra . . . . .                           | 45        |
| 2.3.5 The $SU(m, n)$ Algebra . . . . .                        | 45        |
| 2.4 Disentangling an Exponential . . . . .                    | 48        |
| 2.4.1 The Harmonic Oscillator Algebra . . . . .               | 49        |
| 2.4.2 The $SU(2)$ Algebra . . . . .                           | 50        |

|           |   |           |
|-----------|---|-----------|
| 2.4.3     | <i>SU(1,1)</i> Algebra . . . . .  | 51        |
| 2.5       | Time-Ordered Exponential Integral . . . . .   | 52        |
| 2.5.1     | Harmonic Oscillator Algebra . . . . .   | 52        |
| 2.5.2     | <i>SU(2)</i> Algebra . . . . .  | 53        |
| 2.5.3     | The <i>SU(1,1)</i> Algebra . . . . .  | 53        |
| <b>3.</b> | <b>Representations of Some Lie Algebras . . . . .</b>                               | <b>55</b> |
| 3.1       | Representation by Eigenvectors<br>and Group Parameters . . . . .                    | 55        |
| 3.1.1     | Bases Constituted by Eigenvectors . . . . .   | 55        |
| 3.1.2     | Bases Labeled by Group Parameters . . . . .   | 56        |
| 3.2       | Representations of Harmonic Oscillator Algebra . . . . .                            | 60        |
| 3.2.1     | Orthonormal Bases . . . . .   | 60        |
| 3.2.2     | Minimum Uncertainty Coherent States . . . . .                                       | 61        |
| 3.3       | Representations of <i>SU(2)</i> . . . . .   | 68        |
| 3.3.1     | Orthonormal Representation . . . . .  | 68        |
| 3.3.2     | Minimum Uncertainty Coherent States . . . . .                                       | 70        |
| 3.4       | Representations of <i>SU(1, 1)</i> . . . . .  | 76        |
| 3.4.1     | Orthonormal Bases . . . . .   | 76        |
| 3.4.2     | Minimum Uncertainty Coherent States . . . . .                                       | 77        |
| <b>4.</b> | <b>Quasiprobabilities and Non-classical States . . . . .</b>                        | <b>81</b> |
| 4.1       | Phase Space Distribution Functions . . . . .  | 81        |
| 4.2       | Phase Space Representation of Spins . . . . .                                       | 88        |
| 4.3       | Quasiprobability Distributions for Eigenvalues<br>of Spin Components . . . . .      | 93        |
| 4.4       | Classical and Non-classical States . . . . .  | 95        |
| 4.4.1     | Non-classical States of Electromagnetic Field . . . . .                             | 95        |
| 4.4.2     | Non-classical States of Spin-1/2s . . . . .   | 97        |
| <b>5.</b> | <b>Theory of Stochastic Processes . . . . .</b>                                     | <b>99</b> |
| 5.1       | Probability Distributions . . . . .   | 99        |
| 5.2       | Markov Processes . . . . .  | 102       |
| 5.3       | Detailed Balance . . . . .  | 105       |
| 5.4       | Liouville and Fokker–Planck Equations . . . . .                                     | 106       |
| 5.4.1     | Liouville Equation . . . . .  | 107       |
| 5.4.2     | The Fokker–Planck Equation . . . . .  | 107       |
| 5.5       | Stochastic Differential Equations . . . . .   | 109       |
| 5.6       | Linear Equations with Additive Noise . . . . .                                      | 110       |
| 5.7       | Linear Equations with Multiplicative Noise . . . . .                                | 112       |
| 5.7.1     | Univariate Linear Multiplicative Stochastic Differ-<br>ential Equations . . . . .   | 113       |
| 5.7.2     | Multivariate Linear Multiplicative Stochastic Differ-<br>ential Equations . . . . . | 114       |
| 5.8       | The Poisson Process . . . . .   | 115       |

|           |  |            |
|-----------|--|------------|
| 5.9       | Stochastic Differential Equation<br>Driven by Random Telegraph Noise ..... | 116        |
| <b>6.</b> | <b>The Electromagnetic Field . . . . .</b>                                 | <b>119</b> |
| 6.1       | Free Classical Field .....   | 119        |
| 6.2       | Field Quantization.....  | 121        |
| 6.3       | Statistical Properties of Classical Field .....                            | 123        |
| 6.3.1     | First-Order Correlation Function .....                                     | 125        |
| 6.3.2     | Second-Order Correlation Function .....                                    | 126        |
| 6.3.3     | Higher-Order Correlations .....  | 126        |
| 6.3.4     | Stable and Chaotic Fields .....  | 127        |
| 6.4       | Statistical Properties of Quantized Field.....                             | 130        |
| 6.4.1     | First-Order Correlation .....  | 131        |
| 6.4.2     | Second-Order Correlation .....   | 132        |
| 6.4.3     | Quantized Coherent and Thermal Fields .....                                | 132        |
| 6.5       | Homodyned Detection .....  | 134        |
| 6.6       | Spectrum.....  | 135        |
| <b>7.</b> | <b>Atom–Field Interaction Hamiltonians . . . . .</b>                       | <b>137</b> |
| 7.1       | Dipole Interaction .....   | 137        |
| 7.2       | Rotating Wave and Resonance Approximations.....                            | 140        |
| 7.3       | Two-Level Atom .....   | 144        |
| 7.4       | Three-Level Atom .....   | 145        |
| 7.5       | Effective Two-Level Atom .....   | 146        |
| 7.6       | Multi-channel Models .....   | 149        |
| 7.7       | Parametric Processes .....   | 150        |
| 7.8       | Cavity QED .....   | 151        |
| 7.9       | Moving Atom .....  | 153        |
| <b>8.</b> | <b>Quantum Theory of Damping . . . . .</b>                                 | <b>155</b> |
| 8.1       | The Master Equation .....  | 155        |
| 8.2       | Solving a Master Equation.....   | 160        |
| 8.3       | Multi-Time Average of System Operators.....                                | 162        |
| 8.4       | Bath of Harmonic Oscillators .....   | 163        |
| 8.4.1     | Thermal Reservoir .....  | 164        |
| 8.4.2     | Squeezed Reservoir .....   | 166        |
| 8.4.3     | Reservoir of the Electromagnetic Field .....                               | 167        |
| 8.5       | Master Equation for a Harmonic Oscillator .....                            | 168        |
| 8.6       | Master Equation for Two-Level Atoms .....                                  | 170        |
| 8.6.1     | Two-Level Atom in a Monochromatic Field .....                              | 171        |
| 8.6.2     | Collisional Damping .....  | 172        |
| 8.7       | Master Equation for a Three-Level Atom .....                               | 173        |
| 8.8       | Master Equation for Field Interacting<br>with a Reservoir of Atoms .....   | 174        |

|  |     |
|--|-----|
| <b>9. Linear and Nonlinear Response of a System<br/>in an External Field</b> | 177 |
| 9.1 Steady State of a System in an External Field                            | 177 |
| 9.2 Optical Susceptibility   | 179 |
| 9.3 Rate of Absorption of Energy   | 181 |
| 9.4 Response in a Fluctuating Field  | 183 |
| <b>10. Solution of Linear Equations:<br/>Method of Eigenvector Expansion</b> | 185 |
| 10.1 Eigenvalues and Eigenvectors  | 186 |
| 10.2 Generalized Eigenvalues and Eigenvectors                                | 189 |
| 10.3 Solution of Two-Term Difference-Differential Equation                   | 191 |
| 10.4 Exactly Solvable Two- and Three-Term<br>Recursion Relations             | 192 |
| 10.4.1 Two-Term Recursion Relations  | 192 |
| 10.4.2 Three-Term Recursion Relations  | 193 |
| <b>11. Two-Level<br/>and Three-Level Hamiltonian Systems</b>                 | 199 |
| 11.1 Exactly Solvable Two-Level Systems                                      | 199 |
| 11.1.1 Time-Independent Detuning and Coupling                                | 202 |
| 11.1.2 On-Resonant Real Time-Dependent Coupling                              | 208 |
| 11.1.3 Fluctuating Coupling  | 208 |
| 11.2 $N$ Two-Level Atoms in a Quantized Field                                | 210 |
| 11.3 Exactly Solvable Three-Level Systems                                    | 210 |
| 11.4 Effective Two-Level Approximation                                       | 212 |
| <b>12. Dissipative Atomic Systems</b>  | 215 |
| 12.1 Two-Level Atom in a Quasimonochromatic Field                            | 215 |
| 12.1.1 Time-Dependent Evolution Operator<br>Reducible to $SU(2)$             | 217 |
| 12.1.2 Time-Independent Evolution Operator                                   | 219 |
| 12.1.3 Nonlinear Response in a Bichromatic Field                             | 223 |
| 12.2 $N$ Two-Level Atoms in a Monochromatic Field                            | 224 |
| 12.3 Two-Level Atoms in a Fluctuating Field                                  | 236 |
| 12.4 Driven Three-Level Atom   | 237 |
| <b>13. Dissipative Field Dynamics</b>  | 239 |
| 13.1 Down-Conversion in a Damped Cavity                                      | 239 |
| 13.1.1 Averages and Variances of the Cavity Field Operators                  | 240 |
| 13.1.2 Density Matrix  | 242 |
| 13.2 Field Interacting with a Two-Photon Reservoir                           | 245 |
| 13.2.1 Two-Photon Absorption   | 245 |
| 13.2.2 Two-Photon Generation and Absorption                                  | 247 |
| 13.3 Reservoir in the Lambda Configuration                                   | 248 |

|   |     |
|---|-----|
| <b>14. Dissipative Cavity QED</b>                             | 251 |
| 14.1 Two-Level Atoms in a Single-Mode Cavity                  | 251 |
| 14.2 Strong Atom–Field Coupling                               | 252 |
| 14.2.1 Single Two-Level Atom                                  | 252 |
| 14.3 Response to an External Field                            | 255 |
| 14.3.1 Linear Response to a Monochromatic Field               | 256 |
| 14.3.2 Nonlinear Response to a Bichromatic Field              | 257 |
| 14.4 The Micromaser   | 259 |
| 14.4.1 Density Operator of the Field                          | 259 |
| 14.4.2 Two-Level Atomic Micromaser                            | 263 |
| 14.4.3 Atomic Statistics                                      | 266 |
| <b>Appendices</b>   | 267 |
| A. Some Mathematical Formulae                                 | 267 |
| B. Hypergeometric Equation                                    | 270 |
| C. Solution of Two-<br>and Three-Dimensional Linear Equations | 272 |
| D. Roots of a Polynomial                                      | 273 |
| <b>References</b>   | 277 |
| <b>Index</b>  | 283 |