Contents

Introduction **Dynamic Hedging** 1

Principles of Real World Dynamic Hedging 1
General Risk Management 3

PART I

MARKETS, INSTRUMENTS, PEOPLE

1 **Introduction to the Instruments** 9

Derivatives 9
Synthetic Securities 12
Time-Dependent Linear Derivatives 13
Noncontingent Time-Dependent Nonlinear Derivatives 16
Options and Other Contingent Claims 16
Simple Options 18
Hard and Soft Optionality 20
Basic Rules of Options Equivalence 20
Mirror Image Rule 22
American Options, Early Exercise, and Other Headaches (Advanced Topic) 24
Soft American Options 24
Hard American Options 25
A Brief Warning about Early Exercise Tests 27
Forwards, Futures, and Forward-Forwards (Advanced Topic) 29
Credit 30
Marks-to-Market Differences 30
The Correlation between the Future and the Financing (Advanced Issue) 31
Forward-Forward 32
Core Risk Management: Distinction between Primary and Secondary Risks 32
Applying the Framework to Specific Instruments 35

2 The Generalized Option 38

Step 1. The Homogeneity of the Structure 38
Step 2. The Type of Payoff: Continuous and Discontinuous 41
Step 3. Barriers 43
Step 4. Dimension of the Structure and the Number of Assets 43
Step 5. Order of the Options 45
Step 6. Path Dependence 46

3 Market Making and Market Using 48

Book Runners versus Price Takers 48
Commoditized and Nonstandard Products 50
Trading Risks in Commoditized Products 51
Profitability 53
Proprietary Departments 54
Tacit Rules in Market Making 56
Market Making and the Price for Immediacy 57
Market Making and Autocorrelation of Price Changes 58
Market Making and the Illusion of Profitability 58
Adverse Selection, Signaling, and the Risk Management of Market Makers 60
Value Trading versus the Greater Fool Theory 62
Monkeys on a Typewriter 64
The Statistical Value of Track Records 64
More Modern Methods of Monitoring Traders 65
The Fair Dice and the Dubins-Savage Optimal Strategy 65
The ArcSine Law of the P/L 66

4 Liquidity and Liquidity Holes 68

Liquidity 68
Liquidity Holes 69
Liquidity and Risk Management 70
Stop Orders and the Path of Illiquidity 70
Barrier Options and the Liquidity Vacuum 72
One-Way Liquidity Traps 73
Holes, Black-Scholes, and the Ills of Memory 73
Limits and Market Failures 74
Reverse Slippage 74
Liquidity and Triple Witching Hour 75

Portfolio Insurance 75
Liquidity and Option Pricing 77

5 Arbitrage and the Arbitrageurs **80**

A Trader's Definition 80
Mechanical versus Behavioral Stability 81
The Deterministic Relationships 82
Passive Arbitrage 83
An Absorbing Barrier Called the "Squeeze" 84
Duration of the Arbitrage 84
Arbitrage and the Accounting Systems 85
Other Nonmarket Forms of Arbitrage 86
Arbitrage and the Variance of Returns 87

6 Volatility and Correlation **88**

Calculating Historical Volatility and Correlation 92
Centering around the Mean 92
Introducing Filtering 95
There Is No Such Thing as Constant Volatility and Correlation 97
The Parkinson Number and the Variance Ratio Method 101

PART II

MEASURING OPTION RISKS

The Real World and the Black-Scholes-Merton Assumptions 109
Black-Scholes-Merton as an Almost Nonparametric Pricing System 109

7 Adapting Black-Scholes-Merton: The Delta **115**

Characteristics of a Delta 116
The Continuous Time Delta Is Not Always a Hedge Ratio 116
Delta as a Measure for Risk 121
Confusion: Delta by the Cash or by the Forward 123
Delta for Linear Instruments 123
Delta for a Forward 123
Delta for a Forward-Forward 125
Delta for a Future 125
Delta and the Barrier Options 126

Delta and the Bucketing 127
Delta in the Value at Risk 127
Delta, Volatility, and Extreme Volatility 127

8 **Gamma and Shadow Gamma** **132**

Simple Gamma 132
Gamma Imperfections for a Book 133
Correction for the Gamma of the Back Month 136
First Adjustment 137
Second Adjustment 138
Shadow Gamma 138
Shadow Gamma and the Skew 142
GARCH Gamma 142
Advanced Shadow Gamma 142
Case Study in Shadow Gamma: The Syldavian Elections 145

9 **Vega and the Volatility Surface** **147**

Vega and Modified Vega 147
Vega and the Gamma 149
The Modified Vega 150
How to Compute the Simple Weightings 151
Advanced Method: The Covariance Bucket Vega 153
Forward Implied Volatilities 154
Computing Forward Implied Volatility 154
Multifactor Vega 158
Volatility Surface 164
The Method of Squares for Risk Management 164

10 **Theta and Minor Greeks** **167**

Theta and the Modified Theta 167
Modifying the Theta 167
Theta for a Bet 169
Theta, Interest Carry, and Self-Financing Strategies 169
Shadow Theta 170
Weakness of the Theta Measure 171
Minor Greeks 171
Rho, Modified Rho 171
Omega (Option Duration) 174
Alpha 178

Table of Greeks 181
Stealth and Health 182
Convexity, Modified Convexity 183
The "Double Bubble" 190

***11* The Greeks and Their Behavior** **191**

The Bleed: Gamma and Delta Bleed (Holding Volatility Constant) 191
Bleed with Changes in Volatility 195
Going into the Expiration of a Vanilla Option 196
Ddeltadvol (Stability Ratio) 200
Test 1 of Stability 200
Test 2 of Stability: The Asymptotic Vega Test 201
Moments of an Option Position 202
Ignoring Higher Greeks: The Lock Delta 204

***12* Fungibility, Convergence, and Stacking** **208**

Fungibility 208
Ranking of Fungibility 209
Fungibility and the Term Structure of Prices: The Cash-and-Carry Line 210
Fungibility and Option Arbitrage 212
Changes in the Rules of the Game 212
Convergence 213
Mapping Convergence 215
Convergence and Convexity 216
Levels of Convergence Trading 216
Volatility and Convergence 216
Convergence and Biased Assets 216
Stacking Techniques 217
Other Stacking Applications 220

***13* Some Wrinkles of Option Markets** **222**

Expiration Pin Risks 222
Sticky Strikes 223
Market Barriers 224
A Currency Band: Is It a Barrier? 225
The Absent Barrier 226
What Flat Means 226
Primary and Secondary Exposures 228

14 Bucketing and Topography 229

Static Straight Bucketing 229
American and Path-Dependent Options 231
Advanced Topic: The Forward or "Forward-Forward" Bucket 231
Topography 232
Strike Topography (or Static Topography) 233
Dynamic Topography (Local Volatility Exposure) 235
Barrier Payoff Topography 237

15 Beware the Distribution 238

The Tails 238
Random Volatility 238
Histograms from the Markets 242
The Skew and Biased Assets 245
Biased Assets 248
Nonparallel Accounting 249
Value Linked to Price 250
Currencies as Assets 250
Reverse Assets 251
Volatility Regimes 251
Correlation between Interest Rates and Carry 252
More Advanced Put-Call Parity Rules 252

16 Option Trading Concepts 256

Initiation to Volatility Trading: Vega versus Gamma 260
Soft versus Hard Deltas 262
Volatility Betting 263
Higher Moment Bets 264
Case Study: Path Dependence of a Regular Option 265
Simple Case Study: The "Worst Case" Scenario 270

PART III

TRADING AND HEDGING EXOTIC OPTIONS

17 Binary Options: European Style 273

European Binary Options 273
Hedging with a Vanilla 275
Definition of the Bet: Forward and Spot Bets 278

Pricing with the Skew 279
A Formal Pricing on the Skew 281
The Skew Paradox 282
Difference between the Binary and the Delta: The Delta Paradox Revisited 284
First Hedging Consequences 286
The Delta Is a Dirac Delta 286
Gamma for a Bet 287
Conclusion: Statistical Trading versus Dynamic Hedging 289
Case Study in Binary Packages—Contingent Premium Options 290
Recommended Use: Potential Devaluations 291
Case Study: The Betspreads 292
Advanced Case Study: Multiasset Bets 294

18 **Binary Options: American Style** **295**

American Single Binary Options 295
Hedging an American Binary: Fooled by the Greeks 298
Case Study: National Vega Bank 298
The Ravages of Time 299
Understanding the Vega Convexity 303
Trading Methods 305
Case Study: At-Settlement American Binary Options 306
Other Greeks 307
American Double Binary Options 307
Vegas of the Double Binary 308
Other Applications of American Barriers 309
Credit Risk 311

19 **Barrier Options (I)** **312**

Barrier Options (Regular) 312
Knock-Out Options 312
Knock-In Options 317
Effects of Volatility 319
Adding the Drift: Complexity of the Forward Line 321
Risk Reversals 323
Put/Call Symmetry and the Hedging of Barrier Options 323
Barrier Decomposition under Skew Environments 331
The Reflection Principle 335
Girsanov 339
Effect of Time on Knock-Out Options 339

First Exit Time and Its Risk-Neutral Expectation 340
Issues in Pricing Barrier Options 343
The Single Volatility Fudge 343
A More Accurate Method: The Dupire-Derman-Kani Technique 344
Additional Pricing Complexity: The Variance Ratios 345
Exercise: Adding the Puts 346

20 **Barrier Options (II)** **347**

Reverse Barrier Options 347
Reverse Knock-Out Options 347
Case Study: The Knock-Out Box 348
Hedging Reverse Knock-Outs: A Graphical Case Study 356
Double Barrier Options 362
Rebate 363
Exercise: Adding the Knock-In 363
Alternative Barrier Options 363
The Exploding Option 364
Capped Index Option 365
Reading a Risk Management Report 368
Gaps and Gap Reports 374

21 **Compound, Choosers, and Higher Order Options** **376**

Vega Convexity: The Costs of Dynamic Hedging 378
Uses of Compound Options: Hedging Barrier Vega 379
Chooser Options 380
A Few Applications of the Higher Order Options 382

22 **Multiasset Options** **383**

Choice between Assets: Rainbow Options 384
Correlated and Uncorrelated Greeks 387
Linear Combinations 390
Basket Options 391
Lognormality 391
Correlation Issues 392
Composite Underlying Securities 395
Quantitative Case Study: Indexed Notes 395
Background 396
Terms of the Note 396
Where Is the Underlying? 397
Triangular Decomposition 398

23 Minor Exotics: Lookback and Asian Options **403**

Lookback and Ladder Options 403
The Rollover Option 404
A Footnote on Basket Options: Asian Options 408

PART IV
MODULES

Module A **Brownian Motion on a Spreadsheet, a Tutorial** **415**

The Classical One-Asset Random Walk 415
Some Questions 417
A Two-Asset Random Walk: An Introduction to the Effects of Correlation 420
Extension: A Three-Asset Random Walk 424

Module B **Risk Neutrality Explained** **426**

Step 1. Probabilistic Fairness, the "Fair Dice" and the Skew 426
Step 2. Adding the Real World: The Risk-Neutral Argument 427
The Drift 427

Module C **Numeraire Relativity and the Two-Country Paradox** **431**

Extension: The Two-Country Paradox 433
Conclusion 435
Mathematical Note 436
Conclusion 437

Module D **Correlation Triangles: A Graphical Case Study** **438**

Correlation Triangle Rule 441
Calculating an Implied Correlation Curve 444

Module E **The Value-at-Risk** **445**

Simplified Examples 446
Example 1. No Diversification 447
Example 2. A Cross-Position 447
Example 3. Two Possible Trades 448

Module F **Probabilistic Rankings in Arbitrage** **453**

Ranking of Securities 453
European Option Rules 453
Calendar Rules 454
Barrier and Digital Rules 454
Correlation Rules 455
Correlation Convexity Rules 457
General Convexity Rules 458

Module G **Option Pricing** **459**

Ito's Lemma Explained 459
Ito's Lemma for Two Assets 462
Black-Scholes Equation 463
The Risk-Neutral Argument 463
Stochastic Volatility Model 464
Multiasset Options 466
Rainbow Options 466
Outperformance Options 467
Spread Options 467
Compound and Chooser Order Options 467
Compound Options 468
Chooser Options 468
Barrier Options 468
The Reflection Principle 469
Girsanov's Theorem 469
Pricing Barriers 470
Numerical Stochastic Integration: A Sample 477
A Mathematica™ Program 477

Notes **479**

Bibliography **490**

Index **499**