Contents

List of Figures xvii

Preface xix

1 Finite-Sample Properties of OLS 3

1.1 The Classical Linear Regression Model 3
The Linearity Assumption 4
Matrix Notation 6
The Strict Exogeneity Assumption 7
Implications of Strict Exogeneity 8
Strict Exogeneity in Time-Series Models 9
Other Assumptions of the Model 10
The Classical Regression Model for Random Samples 12
"Fixed" Regressors 13

1.2 The Algebra of Least Squares 15
OLS Minimizes the Sum of Squared Residuals 15
Normal Equations 16
Two Expressions for the OLS Estimator 18
More Concepts and Algebra 18
Influential Analysis (optional) 21
A Note on the Computation of OLS Estimates 23

1.3 Finite-Sample Properties of OLS 27
Finite-Sample Distribution of $\mathbf{b}$ 27
Finite-Sample Properties of s^2 30
Estimate of Var($\mathbf{b} \mid \mathbf{X}$) 31

1.4 Hypothesis Testing under Normality 33
Normally Distributed Error Terms 33
Testing Hypotheses about Individual Regression Coefficients 35
Decision Rule for the t-Test 37
Confidence Interval 38

	p-Value	38
	Linear Hypotheses	39
	The F-Test	40
	A More Convenient Expression for F	42
	t versus F	43
	An Example of a Test Statistic Whose Distribution Depends on $\mathbf{X}$	45
1.5	Relation to Maximum Likelihood	47
	The Maximum Likelihood Principle	47
	Conditional versus Unconditional Likelihood	47
	The Log Likelihood for the Regression Model	48
	ML via Concentrated Likelihood	48
	Cramer-Rao Bound for the Classical Regression Model	49
	The F-Test as a Likelihood Ratio Test	52
	Quasi-Maximum Likelihood	53
1.6	Generalized Least Squares (GLS)	54
	Consequence of Relaxing Assumption 1.4	55
	Efficient Estimation with Known $\mathbf{V}$	55
	A Special Case: Weighted Least Squares (WLS)	58
	Limiting Nature of GLS	58
1.7	Application: Returns to Scale in Electricity Supply	60
	The Electricity Supply Industry	60
	The Data	60
	Why Do We Need Econometrics?	61
	The Cobb-Douglas Technology	62
	How Do We Know Things Are Cobb-Douglas?	63
	Are the OLS Assumptions Satisfied?	64
	Restricted Least Squares	65
	Testing the Homogeneity of the Cost Function	65
	Detour: A Cautionary Note on R^2	67
	Testing Constant Returns to Scale	67
	Importance of Plotting Residuals	68
	Subsequent Developments	68
	Problem Set	71
	Answers to Selected Questions	84
2	**Large-Sample Theory**	**88**
2.1	Review of Limit Theorems for Sequences of Random Variables	88
	Various Modes of Convergence	89
	Three Useful Results	92

	Viewing Estimators as Sequences of Random Variables	94
	Laws of Large Numbers and Central Limit Theorems	95
2.2	Fundamental Concepts in Time-Series Analysis	97
	Need for Ergodic Stationarity	97
	Various Classes of Stochastic Processes	98
	Different Formulation of Lack of Serial Dependence	106
	The CLT for Ergodic Stationary Martingale Differences Sequences	106
2.3	Large-Sample Distribution of the OLS Estimator	109
	The Model	109
	Asymptotic Distribution of the OLS Estimator	113
	s^2 Is Consistent	115
2.4	Hypothesis Testing	117
	Testing Linear Hypotheses	117
	The Test Is Consistent	119
	Asymptotic Power	120
	Testing Nonlinear Hypotheses	121
2.5	Estimating $\mathrm{E}(\varepsilon_i^2 \mathbf{x}_i \mathbf{x}_i')$ Consistently	123
	Using Residuals for the Errors	123
	Data Matrix Representation of $\mathbf{S}$	125
	Finite-Sample Considerations	125
2.6	Implications of Conditional Homoskedasticity	126
	Conditional versus Unconditional Homoskedasticity	126
	Reduction to Finite-Sample Formulas	127
	Large-Sample Distribution of t and F Statistics	128
	Variations of Asymptotic Tests under Conditional Homoskedasticity	129
2.7	Testing Conditional Homoskedasticity	131
2.8	Estimation with Parameterized Conditional Heteroskedasticity (optional)	133
	The Functional Form	133
	WLS with Known $\boldsymbol{\alpha}$	134
	Regression of e_i^2 on $\mathbf{z}_i$ Provides a Consistent Estimate of $\boldsymbol{\alpha}$	135
	WLS with Estimated $\boldsymbol{\alpha}$	136
	OLS versus WLS	137
2.9	Least Squares Projection	137
	Optimally Predicting the Value of the Dependent Variable	138
	Best Linear Predictor	139
	OLS Consistently Estimates the Projection Coefficients	140

2.10 Testing for Serial Correlation 141
Box-Pierce and Ljung-Box 142
Sample Autocorrelations Calculated from Residuals 144
Testing with Predetermined, but Not Strictly Exogenous, Regressors 146
An Auxiliary Regression-Based Test 147
2.11 Application: Rational Expectations Econometrics 150
The Efficient Market Hypotheses 150
Testable Implications 152
Testing for Serial Correlation 153
Is the Nominal Interest Rate the Optimal Predictor? 156
R_t Is Not Strictly Exogenous 158
Subsequent Developments 159
2.12 Time Regressions 160
The Asymptotic Distribution of the OLS Estimator 161
Hypothesis Testing for Time Regressions 163
Appendix 2.A: Asymptotics with Fixed Regressors 164
Appendix 2.B: Proof of Proposition 2.10 165
Problem Set 168
Answers to Selected Questions 183

3 Single-Equation GMM 186
3.1 Endogeneity Bias: Working's Example 187
A Simultaneous Equations Model of Market Equilibrium 187
Endogeneity Bias 188
Observable Supply Shifters 189
3.2 More Examples 193
A Simple Macroeconometric Model 193
Errors-in-Variables 194
Production Function 196
3.3 The General Formulation 198
Regressors and Instruments 198
Identification 200
Order Condition for Identification 202
The Assumption for Asymptotic Normality 202
3.4 Generalized Method of Moments Defined 204
Method of Moments 205
Generalized Method of Moments 206
Sampling Error 207

3.5 Large-Sample Properties of GMM 208
Asymptotic Distribution of the GMM Estimator 209
Estimation of Error Variance 210
Hypothesis Testing 211
Estimation of **S** 212
Efficient GMM Estimator 212
Asymptotic Power 214
Small-Sample Properties 215
3.6 Testing Overidentifying Restrictions 217
Testing Subsets of Orthogonality Conditions 218
3.7 Hypothesis Testing by the Likelihood-Ratio Principle 222
The *LR* Statistic for the Regression Model 223
Variable Addition Test (optional) 224
3.8 Implications of Conditional Homoskedasticity 225
Efficient GMM Becomes 2SLS 226
J Becomes Sargan's Statistic 227
Small-Sample Properties of 2SLS 229
Alternative Derivations of 2SLS 229
When Regressors Are Predetermined 231
Testing a Subset of Orthogonality Conditions 232
Testing Conditional Homoskedasticity 234
Testing for Serial Correlation 234
3.9 Application: Returns from Schooling 236
The NLS-Y Data 236
The Semi-Log Wage Equation 237
Omitted Variable Bias 238
IQ as the Measure of Ability 239
Errors-in-Variables 239
2SLS to Correct for the Bias 242
Subsequent Developments 243
Problem Set 244
Answers to Selected Questions 254

4 Multiple-Equation GMM 258
4.1 The Multiple-Equation Model 259
Linearity 259
Stationarity and Ergodicity 260
Orthogonality Conditions 261
Identification 262

	The Assumption for Asymptotic Normality	264
	Connection to the "Complete" System of Simultaneous Equations	265
4.2	Multiple-Equation GMM Defined	265
4.3	Large-Sample Theory	268
4.4	Single-Equation versus Multiple-Equation Estimation	271
	When Are They "Equivalent"?	272
	Joint Estimation Can Be Hazardous	273
4.5	Special Cases of Multiple-Equation GMM: FIVE, 3SLS, and SUR	274
	Conditional Homoskedasticity	274
	Full-Information Instrumental Variables Efficient (FIVE)	275
	Three-Stage Least Squares (3SLS)	276
	Seemingly Unrelated Regressions (SUR)	279
	SUR versus OLS	281
4.6	Common Coefficients	286
	The Model with Common Coefficients	286
	The GMM Estimator	287
	Imposing Conditional Homoskedasticity	288
	Pooled OLS	290
	Beautifying the Formulas	292
	The Restriction That Isn't	293
4.7	Application: Interrelated Factor Demands	296
	The Translog Cost Function	296
	Factor Shares	297
	Substitution Elasticities	298
	Properties of Cost Functions	299
	Stochastic Specifications	300
	The Nature of Restrictions	301
	Multivariate Regression Subject to Cross-Equation Restrictions	302
	Which Equation to Delete?	304
	Results	305
Problem Set		308
Answers to Selected Questions		320
5	**Panel Data**	**323**
5.1	The Error-Components Model	324
	Error Components	324
	Group Means	327
	A Reparameterization	327
5.2	The Fixed-Effects Estimator	330

The Formula 330
Large-Sample Properties 331
Digression: When η_i Is Spherical 333
Random Effects versus Fixed Effects 334
Relaxing Conditional Homoskedasticity 335
5.3 Unbalanced Panels (optional) 337
"Zeroing Out" Missing Observations 338
Zeroing Out versus Compression 339
No Selectivity Bias 340
5.4 Application: International Differences in Growth Rates 342
Derivation of the Estimation Equation 342
Appending the Error Term 343
Treatment of α_i 344
Consistent Estimation of Speed of Convergence 345
Appendix 5.A: Distribution of Hausman Statistic 346
Problem Set 349
Answers to Selected Questions 363

6 Serial Correlation 365
6.1 Modeling Serial Correlation: Linear Processes 365
MA(q) 366
MA(∞) as a Mean Square Limit 366
Filters 369
Inverting Lag Polynomials 372
6.2 ARMA Processes 375
AR(1) and Its MA(∞) Representation 376
Autocovariances of AR(1) 378
AR(p) and Its MA(∞) Representation 378
ARMA(p, q) 380
ARMA(p, q) with Common Roots 382
Invertibility 383
Autocovariance-Generating Function and the Spectrum 383
6.3 Vector Processes 387
6.4 Estimating Autoregressions 392
Estimation of AR(1) 392
Estimation of AR(p) 393
Choice of Lag Length 394
Estimation of VARs 397
Estimation of ARMA(p, q) 398

6.5 Asymptotics for Sample Means of Serially Correlated Processes 400
LLN for Covariance-Stationary Processes 401
Two Central Limit Theorems 402
Multivariate Extension 404
6.6 Incorporating Serial Correlation in GMM 406
The Model and Asymptotic Results 406
Estimating **S** When Autocovariances Vanish after Finite Lags 407
Using Kernels to Estimate **S** 408
VARHAC 410
6.7 Estimation under Conditional Homoskedasticity (Optional) 413
Kernel-Based Estimation of **S** under Conditional Homoskedasticity 413
Data Matrix Representation of Estimated Long-Run Variance 414
Relation to GLS 415
6.8 Application: Forward Exchange Rates as Optimal Predictors 418
The Market Efficiency Hypothesis 419
Testing Whether the Unconditional Mean Is Zero 420
Regression Tests 423
Problem Set 428
Answers to Selected Questions 441

7 Extremum Estimators 445
7.1 Extremum Estimators 446
"Measurability" of $\hat{\boldsymbol{\theta}}$ 446
Two Classes of Extremum Estimators 447
Maximum Likelihood (ML) 448
Conditional Maximum Likelihood 450
Invariance of ML 452
Nonlinear Least Squares (NLS) 453
Linear and Nonlinear GMM 454
7.2 Consistency 456
Two Consistency Theorems for Extremum Estimators 456
Consistency of M-Estimators 458
Concavity after Reparameterization 461
Identification in NLS and ML 462
Consistency of GMM 467
7.3 Asymptotic Normality 469
Asymptotic Normality of M-Estimators 470
Consistent Asymptotic Variance Estimation 473
Asymptotic Normality of Conditional ML 474

Two Examples 476
Asymptotic Normality of GMM 478
GMM versus ML 481
Expressing the Sampling Error in a Common Format 483
7.4 Hypothesis Testing 487
The Null Hypothesis 487
The Working Assumptions 489
The Wald Statistic 489
The Lagrange Multiplier (LM) Statistic 491
The Likelihood Ratio (LR) Statistic 493
Summary of the Trinity 494
7.5 Numerical Optimization 497
Newton-Raphson 497
Gauss-Newton 498
Writing Newton-Raphson and Gauss-Newton in a Common Format 498
Equations Nonlinear in Parameters Only 499
Problem Set 501
Answers to Selected Questions 505

8 Examples of Maximum Likelihood 507
8.1 Qualitative Response (QR) Models 507
Score and Hessian for Observation *t* 508
Consistency 509
Asymptotic Normality 510
8.2 Truncated Regression Models 511
The Model 511
Truncated Distributions 512
The Likelihood Function 513
Reparameterizing the Likelihood Function 514
Verifying Consistency and Asymptotic Normality 515
Recovering Original Parameters 517
8.3 Censored Regression (Tobit) Models 518
Tobit Likelihood Function 518
Reparameterization 519
8.4 Multivariate Regressions 521
The Multivariate Regression Model Restated 522
The Likelihood Function 523
Maximizing the Likelihood Function 524

Consistency and Asymptotic Normality 525
8.5 FIML 526
The Multiple-Equation Model with Common Instruments Restated 526
The Complete System of Simultaneous Equations 529
Relationship between $(\mathbf{\Gamma}_0, \mathbf{B}_0)$ and $\boldsymbol{\delta}_0$ 530
The FIML Likelihood Function 531
The FIML Concentrated Likelihood Function 532
Testing Overidentifying Restrictions 533
Properties of the FIML Estimator 533
ML Estimation of the SUR Model 535
8.6 LIML 538
LIML Defined 538
Computation of LIML 540
LIML versus 2SLS 542
8.7 Serially Correlated Observations 543
Two Questions 543
Unconditional ML for Dependent Observations 545
ML Estimation of AR(1) Processes 546
Conditional ML Estimation of AR(1) Processes 547
Conditional ML Estimation of AR(p) and VAR(p) Processes 549
Problem Set 551

9 Unit-Root Econometrics 557
9.1 Modeling Trends 557
Integrated Processes 558
Why Is It Important to Know if the Process Is I(1)? 560
Which Should Be Taken as the Null, I(0) or I(1)? 562
Other Approaches to Modeling Trends 563
9.2 Tools for Unit-Root Econometrics 563
Linear I(0) Processes 563
Approximating I(1) by a Random Walk 564
Relation to ARMA Models 566
The Wiener Process 567
A Useful Lemma 570
9.3 Dickey-Fuller Tests 573
The AR(1) Model 573
Deriving the Limiting Distribution under the I(1) Null 574
Incorporating the Intercept 577
Incorporating Time Trend 581

9.4 Augmented Dickey-Fuller Tests 585
The Augmented Autoregression 585
Limiting Distribution of the OLS Estimator 586
Deriving Test Statistics 590
Testing Hypotheses about ζ 591
What to Do When p Is Unknown? 592
A Suggestion for the Choice of $p_{max}(T)$ 594
Including the Intercept in the Regression 595
Incorporating Time Trend 597
Summary of the DF and ADF Tests and Other Unit-Root Tests 599
9.5 Which Unit-Root Test to Use? 601
Local-to-Unity Asymptotics 602
Small-Sample Properties 602
9.6 Application: Purchasing Power Parity 603
The Embarrassing Resiliency of the Random Walk Model? 604
Problem Set 605
Answers to Selected Questions 619

10 Cointegration 623
10.1 Cointegrated Systems 624
Linear Vector I(0) and I(1) Processes 624
The Beveridge-Nelson Decomposition 627
Cointegration Defined 629
10.2 Alternative Representations of Cointegrated Systems 633
Phillips's Triangular Representation 633
VAR and Cointegration 636
The Vector Error-Correction Model (VECM) 638
Johansen's ML Procedure 640
10.3 Testing the Null of No Cointegration 643
Spurious Regressions 643
The Residual-Based Test for Cointegration 644
Testing the Null of Cointegration 649
10.4 Inference on Cointegrating Vectors 650
The SOLS Estimator 650
The Bivariate Example 652
Continuing with the Bivariate Example 653
Allowing for Serial Correlation 654
General Case 657
Other Estimators and Finite-Sample Properties 658

10.5 Application: The Demand for Money in the United States 659
The Data 660
$(m - p, y, R)$ as a Cointegrated System 660
DOLS 662
Unstable Money Demand? 663
Problem Set 665

Appendix A: Partitioned Matrices and Kronecker Products **670**
Addition and Multiplication of Partitioned Matrices 671
Inverting Partitioned Matrices 672

Index **675**

List of Figures

1.1	Hypothetical, True, and Estimated Values	16
1.2	Equipment Investment and Growth	23
1.3	t Distribution	38
1.4	F Distribution	43
1.5	t- versus F-Tests	45
1.6	Output Determination	62
1.7	Plot of Residuals against Log Output	69
2.1	Plots of Serially Uncorrelated Time Series: (a) An i.i.d. $N(0, 1)$ sequence. (b) ARCH(1) with shocks taken from panel (a)	107
2.2	What's Observed When	151
2.3	Inflation and Interest Rates	154
2.4	Real Interest Rates	154
3.1	Supply and Demand Curves	190
4.1	OLS and GMM	283
6.1	Forecast Error, Yen/Dollar	421
6.2	Correlogram of $s30 - f$, Yen/Dollar	422
6.3	Yen/Dollar Spot Rate, Jan. 1975–Dec. 1989	424
6.4	Plot of $s30$ against f, Yen/Dollar	425
6.5	Plot of $s30 - s$ against $f - s$, Yen/Dollar	427
6.6	Correlogram of $s_{t+1} - s_t$, Yen/Dollar	443
6.7	Correlogram of Three-Month Ex-Post Real Rate	443
8.1	Effect of Truncation	512
9.1	Log U.S. Real GDP, Value for 1869 Set to 0	558
9.2	Illustration of the Functional Central Limit Theorem	569
9.3	Finite-Sample Distribution of OLS Estimate of AR(1) Coefficient, $T = 100$	577

9.4 Dollar/Sterling Real Exchange Rate, 1791–1990, 1914 Value Set to 0 605

10.1 (a) Log Income and Consumption 624
10.1 (b) Log Income Minus Log Consumption 649
10.2 U.S. Real NNP and Real M1, in Logs 661
10.3 Log M1 Velocity (left scale) and Short-Term Commercial Paper Rate (right scale) 661
10.4 Plot of $m - p - y$ against R 664