

Contents

Preface xv

Acknowledgments xix

I General Theory 1

1 The Expected Utility Model 3

1.1 Simple and Compound Lotteries 3

1.2 Axioms on Preferences under Uncertainty 4

1.3 The Expected Utility Theorem 6

1.4 Critics of the Expected Utility Model 9

1.4.1 The Allais Paradox 10

1.4.2 The Allais Paradox and Time Consistency 11

1.5 Concluding Remark 14

1.6 Exercises and Extensions 14

2 Risk Aversion 17

2.1 Characterization of Risk Aversion 17

2.2 Comparative Risk Aversion 18

2.3 Certainty Equivalent and Risk Premium 20

2.4 The Arrow-Pratt Approximation 21

2.5 Decreasing Absolute Risk Aversion 24

2.6 Some Classical Utility Functions 25

2.7 Test for Your Own Degree of Risk Aversion 29

2.8 An Application: The Cost of Macroeconomic Risks 32

2.9 Conclusion 34

2.10 Exercises and Extensions 35

3	Change in Risk	39
3.1	The Extremal Approach	40
3.2	Second-Order Stochastic Dominance	42
3.3	Diversification	45
3.4	First-Order Stochastic Dominance	46
3.5	Concluding Remark	47
3.6	Exercises and Extensions	48
II	The Standard Portfolio Problem	51
4	The Standard Portfolio Problem	53
4.1	The Model and Its Basic Properties	53
4.2	The Case of a Small Risk	55
4.3	The Case of HARA Functions	57
4.4	The Impact of Risk Aversion	58
4.5	The Impact of a Change in Risk	59
4.6	Concluding Remark	61
4.7	Exercises and Extensions	62
5	The Equilibrium Price of Risk	65
5.1	A Simple Equilibrium Model for Financial Markets	65
5.2	The Equity Premium Puzzle	68
5.3	The Equity Premium with Limited Participation	71
5.4	The Equity Premium and the Integration of International Financial Markets	73
5.5	Conclusion	75
5.6	Exercises	76
III	Some Technical Tools and Their Applications	79
6	A Hyperplane Separation Theorem	81
6.1	The Diffidence Theorem	81
6.2	Link with the Jensen's Inequality	88
6.3	Applications of the Diffidence Theorem	89
6.3.1	Diffidence	89
6.3.2	Comparative Diffidence	90
6.3.3	Central Risk Aversion	91
6.3.4	Central Riskiness	92

6.4	The Covariance Rule	94
6.5	Conclusion	95
6.6	Exercises and Extensions	96
7	Log-Supermodularity	99
7.1	Definition	99
7.2	Log-Supermodularity and Single Crossing	102
7.2.1	A Theoretical Result	102
7.2.2	Applications to the Standard Portfolio Problem	103
7.2.3	Jewitt's Preference Orders	104
7.3	Expectation of a Log-Supermodular Function	105
7.3.1	A Theoretical Result	105
7.3.2	Two Applications	106
7.4	Concluding Remark	107
7.5	Exercises and Extensions	107
7.6	Appendix	108
IV	Multiple Risks	111
8	Risk Aversion with Background Risk	113
8.1	Preservation of DARA	114
8.2	The Comparative Risk Aversion Is Not Preserved	117
8.3	Extensions with Dependent Background Risk	119
8.3.1	Affiliated Background Risk	119
8.3.2	The Comparative Risk Aversion in the Sense of Ross	121
8.4	Conclusion	123
8.5	Exercises and Extensions	124
9	The Tempering Effect of Background Risk	125
9.1	Risk Vulnerability	126
9.2	Risk Vulnerability and Increase in Risk	130
9.2.1	Increase in Background Risk	130
9.2.2	Increase in the Endogenous Risk	130
9.3	Risk Vulnerability and the Equity Premium Puzzle	131
9.4	Generalized Risk Vulnerability	132
9.5	Standardness	135

- 9.6 Conclusion 138
- 9.7 Exercises and Extensions 139
- 10 Taking Multiple Risks 141
 - 10.1 The Interaction between Asset Demand and Small Gambles 142
 - 10.2 Are Independent Assets Substitutes? 144
 - 10.2.1 The i.i.d. Case 144
 - 10.2.2 The General Case 150
 - 10.3 Conclusion 153
 - 10.4 Exercises and Extensions 153
- 11 The Dynamic Investment Problem 155
 - 11.1 Static versus Dynamic Optimization 157
 - 11.2 The Standard Portfolio Problem 158
 - 11.2.1 The Model 158
 - 11.2.2 The HARA Case 160
 - 11.2.3 A Sufficient Condition for Younger People to Be More Risk-Averse 161
 - 11.3 Discussion of the Results 165
 - 11.3.1 Nonlinear Risk Tolerance 165
 - 11.3.2 Nondifferentiable Marginal Utility 166
 - 11.4 Background Risk and Time Horizon 168
 - 11.4.1 Investors Bear a Background Risk at Retirement 168
 - 11.4.2 Stationary Income Process 171
 - 11.5 Final Remark 172
 - 11.6 Exercises and Extensions 173
- 12 Special Topics in Dynamic Finance 175
 - 12.1 The Length of Periods between Trade 175
 - 12.2 Dynamic Discrete Choice 179
 - 12.3 Constraints on Feasible Strategies 183
 - 12.4 The Effect of a Leverage Constraint 185
 - 12.4.1 The Case of a Lower Bound on the Investment in the Risky Asset 185
 - 12.4.2 The Case of an Upper Bound on the Investment in the Risky Asset 187
 - 12.5 Concluding Remarks 190
 - 12.6 Exercises and Extensions 190

V	The Arrow-Debreu Portfolio Problem	193
13	The Demand for Contingent Claims	195
	13.1 The Model	196
	13.2 Characterization of the Optimal Portfolio	197
	13.3 The Impact of Risk Aversion	200
	13.4 Conclusion	201
	13.5 Exercises and Extensions	202
14	Risk on Wealth	205
	14.1 The Marginal Propensity to Consume in State π	206
	14.2 The Preservation of DARA and IARA	208
	14.3 The Marginal Value of Wealth	210
	14.4 Aversion to Risk on Wealth	211
	14.5 Concluding Remark	212
	14.6 Exercises and Extensions	212
VI	Consumption and Saving	215
15	Consumption under Certainty	217
	15.1 Time Separability	217
	15.2 Exponential Discounting	218
	15.3 Consumption Smoothing under Certainty	219
	15.4 Analogy with the Portfolio Problem	221
	15.5 The Social Cost of Volatility	224
	15.6 The Marginal Propensity to Consume	226
	15.7 Time Diversification and Self-Insurance	227
	15.8 Concluding Remark	232
	15.9 Exercises and Extensions	232
16	Precautionary Saving and Prudence	235
	16.1 Prudence	235
	16.2 The Demand for Saving	239
	16.3 The Marginal Propensity to Consume under Uncertainty	239
	16.3.1 Does Uncertainty Increase the MPC?	240
	16.3.2 Does Uncertainty Make the MPC Decreasing in Wealth?	241
	16.4 More Than Two Periods	242

- 16.4.1 The Euler Equation 242
- 16.4.2 Multiperiod Precautionary Saving 244
- 16.5 Illiquid Saving under Uncertainty 246
- 16.6 Conclusion 247
- 16.7 Exercises and Extensions 248
- 17 The Equilibrium Price of Time 249
 - 17.1 Description of the Economy 250
 - 17.2 The Determinants of the Interest Rate 252
 - 17.2.1 The Interest Rate in the Absence of Growth 252
 - 17.2.2 The Effect of a Sure Growth 253
 - 17.2.3 The Effect of Uncertainty 254
 - 17.3 The Risk-Free Rate Puzzle 256
 - 17.4 The Yield Curve 258
 - 17.4.1 The Pricing Formula 258
 - 17.4.2 The Yield Curve with HARA Utility Functions 260
 - 17.4.3 A Result When There Is No Risk of Recession 261
 - 17.4.4 Exploring the Slope of the Yield Curve When There Is a Risk of Recession 264
 - 17.5 Concluding Remark 267
 - 17.6 Exercises and Extensions 268
- 18 The Liquidity Constraint 269
 - 18.1 Saving as a Buffer Stock 270
 - 18.2 The Liquidity Constraint Raises Risk Aversion 272
 - 18.3 The Liquidity Constraint and the Shape of Absolute Risk Tolerance 273
 - 18.4 Numerical Simulations 277
 - 18.5 Conclusion 279
 - 18.6 Exercises and Extensions 281
- 19 The Saving-Portfolio Problem 285
 - 19.1 Precautionary Saving with an Endogenous Risk 285
 - 19.1.1 The Case of Complete Markets 285
 - 19.1.2 The Case of the Standard Portfolio Problem 287
 - 19.1.3 Discussion of the Results 288

19.2	Optimal Portfolio Strategy with Consumption	290
19.3	The Merton-Samuelson Model	291
19.4	Concluding Remark	295
19.5	Exercises and Extensions	295
20	Disentangling Risk and Time	297
20.1	The Model of Kreps and Porteus	298
20.2	Preferences for an Early Resolution of Uncertainty	299
20.3	Prudence with Kreps-Porteus Preferences	300
20.4	Conclusion	302
20.5	Exercises and Extensions	303
VII	Equilibrium Prices of Risk and Time	305
21	Efficient Risk Sharing	307
21.1	The Case of a Static Exchange Economy	307
21.2	The Mutuality Principle	309
21.3	The Sharing of the Social Risk	311
21.3.1	Decomposition of the Problem	311
21.3.2	The Veil of Ignorance	312
21.3.3	Efficient Sharing Rules of the Macro Risk	312
21.3.4	A Two-Fund Separation Theorem	314
21.3.5	The Case of Small Risk per Capita	315
21.4	Group's Attitude toward Risk	316
21.4.1	The Representative Agent	316
21.4.2	Arrow-Lind Theorem	317
21.4.3	Group Decision and Individual Choice	317
21.5	Introducing Time and Investment	319
21.6	A Final Remark: The Concavity of the Certainty Equivalent Functional	321
21.7	Conclusion	323
21.8	Exercises and Extensions	323
21.9	Appendix	325
22	The Equilibrium Price of Risk and Time	327
22.1	An Arrow-Debreu Economy	327
22.2	Application of the First Theorem of Welfare Economics	328
22.3	Pricing Arrow-Debreu Securities	329

22.4	Pricing by Arbitrage	330
22.5	The Competitive Price of Risk	332
22.6	The Competitive Price of Time	334
22.7	Spot Markets and Markets for Futures	335
22.8	Corporate Finance in an Arrow-Debreu Economy	337
22.9	Conclusion	339
22.10	Exercises and Extensions	340
23	Searching for the Representative Agent	343
23.1	Analytical Solution to the Aggregation Problem	344
23.2	Wealth Inequality, Risk Aversion, and the Equity Premium	345
23.3	Wealth Inequality and the Risk-Free Rate	347
23.3.1	The Consumption Smoothing Effect	348
23.3.2	The Precautionary Effect	349
23.4	Conclusion	351
23.5	Exercises and Extensions	352
VIII Risk and Information		355
24	The Value of Information	357
24.1	The General Model of Risk and Information	357
24.1.1	Structure of Information	357
24.1.2	The Decision Problem	358
24.1.3	The Posterior Maximum Expected Utility Is Convex in the Vector of Posterior Probabilities	359
24.2	The Value of Information Is Positive	362
24.3	Refining the Information Structure	364
24.3.1	Definition and Basic Characterization	364
24.3.2	Garbling Messages and the Theorem of Blackwell	366
24.3.3	Location Experiments	371
24.4	The Value of Information and Risk Aversion	373
24.4.1	A Definition of the Value of Information	373
24.4.2	A Simple Illustration: The Gambler's Problem	374
24.4.3	The Standard Portfolio Problem	378
24.5	Conclusion	379

24.6	Exercises and Extensions	380
24.7	Appendix	382
25	Decision Making and Information	383
25.1	A Technique for the Comparative Statics of More Informativeness	383
25.2	The Portfolio-Saving Problem	386
25.3	A Digression: Scientific Uncertainty, Global Warming, and the "Precautionary Principle"	389
25.4	The Saving Problem with Uncertain Returns	390
25.5	Precautionary Saving	392
25.6	The Value of Flexibility and Option Value	393
25.7	Predictability and Portfolio Management	397
	25.7.1 Exogenous Predictability	399
	25.7.2 Endogenous Predictability and Mean-Reversion	400
25.8	Conclusion	405
25.9	Exercises and Extensions	405
26	Information and Equilibrium	407
26.1	Hirshleifer Effect	407
26.2	Information and the Equity Premium	413
26.3	Conclusion	418
26.4	Exercises and Extensions	418
26.5	Appendix	420
27	Epilogue	423
27.1	The Important Open Questions	423
	27.1.1 The Independence Axiom	423
	27.1.2 Measures of Risk Aversion	424
	27.1.3 Qualitative Properties of the Utility Function	425
	27.1.4 Economics of Uncertainty and Psychology	426
27.2	Conclusion	427
	Bibliography	429
	Index of Lemmas and Propositions	441
	Index of Subjects	443