

# Contents

<b>Preface</b>	<b>ix</b>
<b>0 Trigonometry</b>	<b>1</b>
1 What is new about trigonometry? . . . . .	1
2 Right triangles . . . . .	6
3 The Pythagorean theorem . . . . .	7
4 Our best friends (among right triangles) . . . . .	9
5 Our next best friends (among right triangles) . . . . .	10
6 Some standard notation . . . . .	12
<b>1 Trigonometric Ratios in a Triangle</b>	<b>21</b>
1 Definition of $\sin \alpha$ . . . . .	21
2 Find the hidden sine . . . . .	24
3 The cosine ratio . . . . .	26
4 A relation between the sine and the cosine . . . . .	28
5 A bit of notation . . . . .	28
6 Another relation between the sine and the cosine . . . . .	29
7 Our next best friends (and the sine ratio) . . . . .	30
8 What is the value of $\sin 90^\circ$ ? . . . . .	32
9 An exploration: How large can the sum be? . . . . .	33
10 More exploration: How large can the product be? . . . . .	34
11 More names for ratios . . . . .	35
<b>2 Relations among Trigonometric Ratios</b>	<b>41</b>
1 The sine and its relatives . . . . .	41
2 Algebra or geometry? . . . . .	44
3 A remark about names . . . . .	45
4 An identity crisis? . . . . .	46
5 Identities with secant and cosecant . . . . .	48
6 A lemma . . . . .	50
7 Some inequalities . . . . .	51

8	Calculators and tables . . . . .	52
9	Getting the degree measure of an angle from its sine . . . . .	53
10	Solving right triangles . . . . .	55
11	Shadows . . . . .	56
12	Another approach to the sine ratio . . . . .	57
<b>3</b>	<b>Relationships in a Triangle</b>	<b>67</b>
1	Geometry of the triangle . . . . .	67
2	The congruence theorems and trigonometry . . . . .	68
3	Sines and altitudes . . . . .	69
4	Obtuse triangles . . . . .	70
5	The Law of Sines . . . . .	71
6	The circumradius . . . . .	74
7	Area of a triangle . . . . .	75
8	Two remarks . . . . .	78
9	Law of cosines . . . . .	78
<b>4</b>	<b>Angles and Rotations</b>	<b>91</b>
1	Measuring rotations . . . . .	91
2	Rotation and angles . . . . .	93
3	Trigonometric functions for all angles . . . . .	94
4	Calculations with angles of rotations . . . . .	98
5	Odd and even functions . . . . .	101
<b>5</b>	<b>Radian Measure</b>	<b>103</b>
1	Radian measure for angles and rotations . . . . .	103
2	Radian measure and distance . . . . .	109
3	Interlude: How to explain radian measure to your brother and sister . . . . .	113
4	Radian measure and calculators . . . . .	114
5	An important graph . . . . .	115
6	Two small miracles . . . . .	117
<b>6</b>	<b>The Addition Formulas</b>	<b>123</b>
1	More identities . . . . .	123
2	The addition formulas . . . . .	125
3	Proofs of the addition formulas . . . . .	126
4	A first beautiful proof . . . . .	127
5	A second beautiful proof . . . . .	130

<b>7</b>	<b>Trigonometric Identities</b>	<b>139</b>
1	Extending the identities . . . . .	139
2	The Principle of Analytic Continuation: Higher mathematics to the rescue . . . . .	139
3	Back to our identities . . . . .	141
4	A formula for $\tan(\alpha + \beta)$ . . . . .	143
5	Double the angle . . . . .	145
6	Triple the angle . . . . .	148
7	Derivation of the formulas for $\sin \alpha/2$ and $\cos \alpha/2$ . . . . .	149
8	Another formula for $\tan \alpha/2$ . . . . .	151
9	Products to sums . . . . .	152
10	Sums to products . . . . .	154
<b>8</b>	<b>Graphs of Trigonometric Functions</b>	<b>173</b>
1	Graphing the basic sine curve . . . . .	173
2	The period of the function $y = \sin x$ . . . . .	175
3	Periods of other sinusoidal curves . . . . .	176
4	The amplitude of a sinusoidal curve . . . . .	178
5	Shifting the sine . . . . .	179
6	Shifting and stretching . . . . .	182
7	Some special shifts: Half-periods . . . . .	183
8	Graphing the tangent and cotangent functions . . . . .	187
9	An important question about sums of sinusoidal functions . . . . .	188
10	Linear combinations of sines and cosines . . . . .	189
11	Linear combinations of sinusoidal curves with the same frequency . . . . .	192
12	Linear combinations of functions with different frequencies . . . . .	194
13	Finding the period of a sum of sinusoidal curves with different periods . . . . .	196
14	A discovery of Monsieur Fourier . . . . .	197
<b>9</b>	<b>Inverse Functions and Trigonometric Equations</b>	<b>207</b>
1	Functions and Inverse Functions . . . . .	207
2	Arcsin: The inverse function to $\sin$ . . . . .	209
3	Graphing inverse functions . . . . .	214
4	Trigonometric equations . . . . .	217
5	A more general trigonometric equation . . . . .	220
6	More complicated trigonometric equations . . . . .	222