

Contents

PREFACE TO THE FOURTH EDITION	xi
PROLOGUE TO INTRODUCTION TO MATHEMATICAL FINANCE	xiii
1 SET	1
1.1 Sample sets	1
1.2 Operations with sets	3
1.3 Various relations	7
1.4 Indicator	13
Exercises	17
2 PROBABILITY	20
2.1 Examples of probability	20
2.2 Definition and illustrations	24
2.3 Deductions from the axioms	31
2.4 Independent events	35
2.5 Arithmetical density	39
Exercises	42
3 COUNTING	46
3.1 Fundamental rule	46
3.2 Diverse ways of sampling	49
3.3 Allocation models; binomial coefficients	55
3.4 How to solve it	62
Exercises	70
	vii

4	RANDOM VARIABLES	74
4.1	What is a random variable?	74
4.2	How do random variables come about?	78
4.3	Distribution and expectation	84
4.4	Integer-valued random variables	90
4.5	Random variables with densities	95
4.6	General case	105
	Exercises	109
	APPENDIX 1: BOREL FIELDS AND GENERAL RANDOM VARIABLES	115
5	CONDITIONING AND INDEPENDENCE	117
5.1	Examples of conditioning	117
5.2	Basic formulas	122
5.3	Sequential sampling	131
5.4	Pólya's urn scheme	136
5.5	Independence and relevance	141
5.6	Genetical models	152
	Exercises	157
6	MEAN, VARIANCE, AND TRANSFORMS	164
6.1	Basic properties of expectation	164
6.2	The density case	169
6.3	Multiplication theorem; variance and covariance	173
6.4	Multinomial distribution	180
6.5	Generating function and the like	187
	Exercises	195
7	POISSON AND NORMAL DISTRIBUTIONS	203
7.1	Models for Poisson distribution	203
7.2	Poisson process	211
7.3	From binomial to normal	222
7.4	Normal distribution	229
7.5	Central limit theorem	233
7.6	Law of large numbers	239
	Exercises	246
	APPENDIX 2: STIRLING'S FORMULA AND DE MOIVRE-LAPLACE'S THEOREM	251

8	FROM RANDOM WALKS TO MARKOV CHAINS	254
8.1	Problems of the wanderer or gambler	254
8.2	Limiting schemes	261
8.3	Transition probabilities	266
8.4	Basic structure of Markov chains	275
8.5	Further developments	284
8.6	Steady state	291
8.7	Winding up (or down?)	303
	Exercises	314
	APPENDIX 3: MARTINGALE	325
9	MEAN-VARIANCE PRICING MODEL	329
9.1	An investments primer	329
9.2	Asset return and risk	331
9.3	Portfolio allocation	335
9.4	Diversification	336
9.5	Mean-variance optimization	337
9.6	Asset return distributions	346
9.7	Stable probability distributions	348
	Exercises	351
	APPENDIX 4: PARETO AND STABLE LAWS	355
10	OPTION PRICING THEORY	359
10.1	Options basics	359
10.2	Arbitrage-free pricing: 1-period model	366
10.3	Arbitrage-free pricing: N -period model	372
10.4	Fundamental asset pricing theorems	376
	Exercises	377
	GENERAL REFERENCES	379
	ANSWERS TO PROBLEMS	381
	VALUES OF THE STANDARD NORMAL DISTRIBUTION FUNCTION	393
	INDEX	397