Contents

Preface to the Second Edition xi

Preface to the First Edition xiii

1. Probability 1

1.1 Introduction, 1
1.2 Sample Space, 2
1.3 Probability Axioms, 7
1.4 Combinatorics: Probability on Finite Sample Spaces, 21
1.5 Conditional Probability and Bayes Theorem, 28
1.6 Independence of Events, 33

2. Random Variables and Their Probability Distributions 40

2.1 Introduction, 40
2.2 Random Variables, 40
2.3 Probability Distribution of a Random Variable, 43
2.4 Discrete and Continuous Random Variables, 48
2.5 Functions of a Random Variable, 57

3. Moments and Generating Functions 69

3.1 Introduction, 69
3.2 Moments of a Distribution Function, 69
3.3 Generating Functions, 85
3.4 Some Moment Inequalities, 95

4. Multiple Random Variables 102

4.1 Introduction, 102

4.2 Multiple Random Variables, 102
4.3 Independent Random Variables, 119
4.4 Functions of Several Random Variables, 127
4.5 Covariance, Correlation, and Moments, 149
4.6 Conditional Expectation, 164
4.7 Order Statistics and Their Distributions, 171

5. Some Special Distributions **180**

5.1 Introduction, 180
5.2 Some Discrete Distributions, 180
5.3 Some Continuous Distributions, 204
5.4 Bivariate and Multivariate Normal Distributions, 238
5.5 Exponential Family of Distributions, 251

6. Limit Theorems **256**

6.1 Introduction, 256
6.2 Modes of Convergence, 256
6.3 Weak Law of Large Numbers, 274
6.4 Strong Law of Large Numbers, 281
6.5 Limiting Moment Generating Functions, 289
6.6 Central Limit Theorem, 293

7. Sample Moments and Their Distributions **306**

7.1 Introduction, 306
7.2 Random Sampling, 307
7.3 Sample Characteristics and Their Distributions, 310
7.4 Chi-Square, t-, and F-Distributions: Exact Sampling Distributions, 324
7.5 Large-Sample Theory, 334
7.6 Distribution of $(\bar{X}, S^2)$ in Sampling from a Normal Population, 339
7.7 Sampling from a Bivariate Normal Distribution, 344

8. Parametric Point Estimation **353**

8.1 Introduction, 353
8.2 Problem of Point Estimation, 354
8.3 Sufficiency, Completeness, and Ancillarity, 358
8.4 Unbiased Estimation, 377
8.5 Unbiased Estimation (*Continued*): Lower Bound for the Variance of an Estimator, 391

8.6 Substitution Principle (Method of Moments), 406
8.7 Maximum Likelihood Estimators, 409
8.8 Bayes and Minimax Estimation, 424
8.9 Principle of Equivariance, 442

9. Neyman–Pearson Theory of Testing of Hypotheses **454**

9.1 Introduction, 454
9.2 Some Fundamental Notions of Hypotheses Testing, 454
9.3 Neyman–Pearson Lemma, 464
9.4 Families with Monotone Likelihood Ratio, 472
9.5 Unbiased and Invariant Tests, 479
9.6 Locally Most Powerful Tests, 486

10. Some Further Results of Hypothesis Testing **490**

10.1 Introduction, 490
10.2 Generalized Likelihood Ratio Tests, 490
10.3 Chi-Square Tests, 500
10.4 t-Tests, 512
10.5 F-Tests, 518
10.6 Bayes and Minimax Procedures, 520

11. Confidence Estimation **527**

11.1 Introduction, 527
11.2 Some Fundamental Notions of Confidence Estimation, 527
11.3 Methods of Finding Confidence Intervals, 532
11.4 Shortest-Length Confidence Intervals, 546
11.5 Unbiased and Equivariant Confidence Intervals, 553

12. General Linear Hypothesis **561**

12.1 Introduction, 561
12.2 General Linear Hypothesis, 561
12.3 Regression Model, 569
12.4 One-Way Analysis of Variance, 577
12.5 Two-Way Analysis of Variance with One Observation per Cell, 583
12.6 Two-Way Analysis of Variance with Interaction, 590

13. Nonparametric Statistical Inference **598**

13.1 Introduction, 598

13.2 U-Statistics, 598
13.3 Some Single-Sample Problems, 608
13.4 Some Two-Sample Problems, 624
13.5 Tests of Independence, 633
13.6 Some Applications of Order Statistics, 644
13.7 Robustness, 650

References **663**

Frequently Used Symbols and Abbreviations **669**

Statistical Tables **673**

Answers to Selected Problems **693**

Author Index **705**

Subject Index **707**