

Contents

| | |
|--|----|
| 1 The Mathematics of Compound Interest | |
| 1.1 Mathematical Bases of Life Contingencies | 1 |
| 1.2 Effective Interest Rates | 1 |
| 1.3 Nominal Interest Rates | 2 |
| 1.4 Continuous Payments | 3 |
| 1.5 Interest in Advance | 4 |
| 1.6 Perpetuities | 6 |
| 1.7 Annuities | 9 |
| 1.8 Repayment of a Debt | 11 |
| 1.9 Internal Rate of Return | 13 |
| 2 The Future Lifetime of a Life Aged x | |
| 2.1 The Model | 15 |
| 2.2 The Force of Mortality | 16 |
| 2.3 Analytical Distributions of T | 17 |
| 2.4 The Curtate Future Lifetime of (x) | 18 |
| 2.5 Life Tables | 20 |
| 2.6 Probabilities of Death for Fractions of a Year | 21 |
| 3 Life Insurance | |
| 3.1 Introduction | 23 |
| 3.2 Elementary Insurance Types | 23 |
| 3.2.1 Whole Life and Term Insurance | 23 |
| 3.2.2 Pure Endowments | 24 |
| 3.2.3 Endowments | 25 |
| 3.3 Insurances Payable at the Moment of Death | 26 |
| 3.4 General Types of Life Insurance | 27 |
| 3.5 Standard Types of Variable Life Insurance | 29 |
| 3.6 Recursive Formulae | 31 |
| 4 Life Annuities | |
| 4.1 Introduction | 35 |
| 4.2 Elementary Life Annuities | 35 |

| | | |
|----------|---|----|
| 4.3 | Payments made more Frequently than Once a Year | 37 |
| 4.4 | Variable Life Annuities | 39 |
| 4.5 | Standard Types of Life Annuity | 41 |
| 4.6 | Recursion Formulae | 42 |
| 4.7 | Inequalities | 43 |
| 4.8 | Payments Starting at Non-integral Ages | 46 |
| 5 | Net Premiums | |
| 5.1 | Introduction | 49 |
| 5.2 | An Example | 49 |
| 5.3 | Elementary Forms of Insurance | 52 |
| 5.3.1 | Whole Life and Term Insurance | 52 |
| 5.3.2 | Pure Endowments | 53 |
| 5.3.3 | Endowments | 54 |
| 5.3.4 | Deferred Life Annuities | 54 |
| 5.4 | Premiums Paid m Times a Year | 54 |
| 5.5 | A General Type of Life Insurance | 55 |
| 5.6 | Policies with Premium Refund | 56 |
| 5.7 | Stochastic Interest | 56 |
| 6 | Net Premium Reserves | |
| 6.1 | Introduction | 59 |
| 6.2 | Two Examples | 59 |
| 6.3 | Recursive Considerations | 61 |
| 6.4 | The Survival Risk | 63 |
| 6.5 | The Net Premium Reserve of a Whole Life Insurance | 63 |
| 6.6 | Net Premium Reserves at Fractional Durations | 64 |
| 6.7 | Allocation of the Overall Loss to Policy Years | 65 |
| 6.8 | Conversion of an Insurance | 68 |
| 6.9 | Technical Gain | 69 |
| 6.10 | Procedure for Pure Endowments | 70 |
| 6.11 | The Continuous Model | 71 |
| 7 | Multiple Decremnts | |
| 7.1 | The Model | 75 |
| 7.2 | Forces of Decrement | 76 |
| 7.3 | The Curtate Lifetime of (x) | 76 |
| 7.4 | A General Type of Insurance | 77 |
| 7.5 | The Net Premium Reserve | 78 |
| 7.6 | The Continuous Model | 80 |

| | |
|--|-----|
| 8 Multiple Life Insurance | |
| 8.1 Introduction | 83 |
| 8.2 The Joint-Life Status | 83 |
| 8.3 Simplifications | 84 |
| 8.4 The Last-Survivor Status | 85 |
| 8.5 The General Symmetric Status | 87 |
| 8.6 The Schuette-Nesbitt Formula | 89 |
| 8.7 Asymmetric Annuities | 90 |
| 8.8 Asymmetric Insurances | 91 |
| 9 The Total Claim Amount in a Portfolio | |
| 9.1 Introduction | 93 |
| 9.2 The Normal Approximation | 93 |
| 9.3 Exact Calculation of the Total Claim Amount Distribution | 94 |
| 9.4 The Compound Poisson Approximation | 96 |
| 9.5 Recursive Calculation of the Compound Poisson Distribution | 98 |
| 9.6 Reinsurance | 100 |
| 9.7 Stop-Loss Reinsurance | 101 |
| 10 Expense Loadings | |
| 10.1 Introduction | 103 |
| 10.2 The Expense-Loaded Premium | 104 |
| 10.3 Expense-Loaded Premium Reserves | 105 |
| 11 Estimating Probabilities of Death | |
| 11.1 Problem Description | 109 |
| 11.2 The Classical Method | 110 |
| 11.3 Alternative Solution | 111 |
| 11.4 The Maximum Likelihood Method | 112 |
| 11.5 Statistical Inference | 112 |
| 11.6 The Bayesian Approach | 116 |
| 11.7 Multiple Causes of Decrement | 116 |
| 11.8 Interpretation of Results | 118 |
| Appendix A. Commutation Functions | |
| A.1 Introduction | 119 |
| A.2 The Deterministic Model | 119 |
| A.3 Life Annuities | 120 |
| A.4 Life Insurance | 121 |
| A.5 Net Annual Premiums and Premium Reserves | 122 |
| Appendix B. Simple Interest | 125 |

Appendix C. Exercises

| | | |
|--------|--|-----|
| C.0 | Introduction | 128 |
| C.1 | Mathematics of Compound Interest: Exercises | 129 |
| C.1.1 | Theory Exercises | 129 |
| C.1.2 | Spreadsheet Exercises | 130 |
| C.2 | The Future Lifetime of a Life Aged x : Exercises | 133 |
| C.2.1 | Theory Exercises | 134 |
| C.2.2 | Spreadsheet Exercises | 136 |
| C.3 | Life Insurance | 138 |
| C.3.1 | Theory Exercises | 138 |
| C.3.2 | Spreadsheet Exercises | 141 |
| C.4 | Life Annuities | 142 |
| C.4.1 | Theory Exercises | 142 |
| C.4.2 | Spreadsheet Exercises | 144 |
| C.5 | Net Premiums | 146 |
| C.5.1 | Notes | 146 |
| C.5.2 | Theory Exercises | 146 |
| C.5.3 | Spreadsheet Exercises | 150 |
| C.6 | Net Premium Reserves | 152 |
| C.6.1 | Theory Exercises | 152 |
| C.6.2 | Spreadsheet Exercises | 155 |
| C.7 | Multiple Decremnts: Exercises | 156 |
| C.7.1 | Theory Exercises | 156 |
| C.8 | Multiple Life Insurance: Exercises | 158 |
| C.8.1 | Theory Exercises | 158 |
| C.8.2 | Spreadsheet Exercises | 160 |
| C.9 | The Total Claim Amount in a Portfolio | 161 |
| C.9.1 | Theory Exercises | 161 |
| C.10 | Expense Loadings | 163 |
| C.10.1 | Theory Exercises | 163 |
| C.10.2 | Spreadsheet Exercises | 164 |
| C.11 | Estimating Probabilities of Death | 165 |
| C.11.1 | Theory Exercises | 165 |

Appendix D. Solutions

| | | |
|-------|--|-----|
| D.0 | Introduction | 168 |
| D.1 | Mathematics of Compound Interest | 169 |
| D.1.1 | Solutions to Theory Exercises | 169 |
| D.1.2 | Solutions to Spreadsheet Exercises | 170 |
| D.2 | The Future Lifetime of a Life Aged x | 172 |
| D.2.1 | Solutions to Theory Exercise | 172 |
| D.2.2 | Solutions to Spreadsheet Exercises | 174 |
| D.3 | Life Insurance | 175 |
| D.3.1 | Solutions to Theory Exercises | 175 |
| D.3.2 | Solution to Spreadsheet Exercises | 178 |

| | | |
|-----------------------------|---|-----|
| D.4 | Life Annuities | 179 |
| D.4.1 | Solutions to Theory Exercises | 179 |
| D.4.2 | Solutions to Spreadsheet Exercises | 181 |
| D.5 | Net Premiums: Solutions | 183 |
| D.5.1 | Theory Exercises | 183 |
| D.5.2 | Solutions to Spreadsheet Exercises | 187 |
| D.6 | Net Premium Reserves: Solutions | 189 |
| D.6.1 | Theory Exercises | 189 |
| D.6.2 | Solutions to Spreadsheet Exercises | 190 |
| D.7 | Multiple Decremnts: Solutions | 192 |
| D.7.1 | Theory Exercises | 192 |
| D.8 | Multiple Life Insurance: Solutions | 194 |
| D.8.1 | Theory Exercises | 194 |
| D.8.2 | Solutions to Spreadsheet Exercises | 197 |
| D.9 | The Total Claim Amount in a Portfolio | 198 |
| D.9.1 | Theory Exercises | 198 |
| D.10 | Expense Loadings | 200 |
| D.10.1 | Theory Exercises | 200 |
| D.10.2 | Spreadsheet Exercises | 201 |
| D.11 | Estimating Probabilities of Death | 203 |
| D.11.1 | Theory Exercises | 203 |
| Appendix E. Tables | | |
| E.0 | Illustrative Life Tables | 207 |
| E.1 | Commutation Columns | 209 |
| E.2 | Multiple Decrement Tables | 211 |
| References 213 | | |
| Index 215 | | |