Chapter 0

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As this book is long and quite complicated, the reader is recommended to study this table in more detail than might normally be considered, in order to appraise the general structure and order of the material. The title of every section and sub-section is given here, and the contents of each chapter is repeated at its head.

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