

Contents

| | | |
|--------|---|----|
| | Introduction | 1 |
| 1. | The Early Work of Charles S. Peirce | 9 |
| 1.1. | Overview of the Mathematical Systems of Charles S. Peirce | 9 |
| 1.2. | Peirce's Influence on the Development of Logic | 11 |
| 1.3. | Peirce's Early Approaches to Logic | 14 |
| 2. | Peirce's Calculus of Relatives: 1870 | 23 |
| 2.1. | Peirce's Algebra of Relations | 24 |
| 2.1.1. | Inclusion and Equality | 27 |
| 2.1.2. | Addition | 29 |
| 2.1.3. | Multiplication | 30 |
| 2.1.4. | Peirce's First Quantifiers | 38 |
| 2.1.5. | Involution | 39 |
| 2.1.6. | Involution and Mixed-Quantifier Forms | 42 |
| 2.1.7. | Elementary Relatives | 44 |
| 2.2. | Quantification in the Calculus of Relatives in 1870 | 46 |
| 2.3. | Summary | 48 |
| 3. | Peirce on the Algebra of Logic: 1880 | 51 |
| 3.1. | Overview of Peirce's "On the algebra of logic" | 51 |
| 3.2. | Discussion | 54 |
| 3.2.1. | The Origins of Logic | 54 |
| 3.2.2. | Syllogism and Illation | 56 |
| 3.2.3. | Forms of Propositions | 60 |
| 3.2.4. | The Algebra of the Copula | 64 |
| 3.2.5. | The Logic of Nonrelative Terms | 70 |
| 3.3. | Conclusion | 73 |
| 4. | Mitchell on a New Algebra of Logic: 1883 | 75 |
| 4.1. | Mitchell's Rule of Inference | 75 |
| 4.2. | Single-Variable Monadic Logic | 78 |

| | | |
|--------|--|-----|
| 4.2.1. | Single-Variable Monadic Propositions | 78 |
| 4.2.2. | Disjunctive Normal Form | 80 |
| 4.2.3. | Rules of Inference for Single-Variable Logic | 82 |
| 4.3. | Two-Variable Monadic Logic | 86 |
| 4.3.1. | Mitchell's Dimension Theory | 86 |
| 4.3.2. | Contrast to Peirce | 88 |
| 4.4. | Three-Variable Monadic Logic | 89 |
| 4.5. | Peirce on Mitchell | 90 |
| 5. | Peirce on the Algebra of Relatives: 1883 | 95 |
| 5.1. | Background in Linear Associative Algebras | 95 |
| 5.2. | The Algebra of Relatives | 98 |
| 5.2.1. | Types of Relatives | 98 |
| 5.2.2. | Operations on Relatives | 100 |
| 5.3. | Syllogistic in the Relative Calculus | 104 |
| 5.4. | Prenex Predicate Calculus | 106 |
| 5.5. | Summary of Peirce's Accomplishments in 1883 | 109 |
| 5.5.1. | Syntax and Semantics | 109 |
| 5.5.2. | Quantifiers | 110 |
| 5.6. | Peirce's Appraisal of His Algebra of Binary Relatives | 111 |
| 6. | Peirce's Logic of Quantifiers: 1885 | 113 |
| 6.1. | On the Derivation of Logic from Algebra | 113 |
| 6.2. | Nonrelative Logic | 116 |
| 6.2.1. | Embedding Boolean Algebra in Ordinary Algebra | 116 |
| 6.2.2. | Five Peirce Icons | 121 |
| 6.2.3. | Truth-functional Interpretations of Propositions | 125 |
| 6.3. | First-Order Logic | 127 |
| 6.3.1. | Infinite Sums and Products | 127 |
| 6.3.2. | Mitchell | 128 |
| 6.3.3. | Formulas and Rules | 129 |
| 6.4. | Second-Order Logic | 132 |
| 7. | Schröder's Calculus of Relatives | 143 |
| 7.1. | <i>Die Algebra der Logik</i> : Volume 1 | 144 |
| 7.2. | <i>Die Algebra der Logik</i> : Volume 2 | 147 |
| 7.3. | <i>Die Algebra der Logik</i> : Volume 3 | 149 |
| 7.3.1. | Peirce's Attack on the General Solutions of Schröder | 153 |
| 7.3.2. | Lectures VI–X and Dedekind Chain Theory | 155 |
| 7.3.3. | Lectures XI–XII and Higher Order Logic | 160 |
| 7.4. | Norbert Wiener's Ph.D. Thesis | 165 |
| 8. | Löwenheim's Contribution | 169 |
| 8.1. | Overview of Löwenheim's 1915 Paper | 171 |

| | | |
|------|-----------------------------|-----|
| 8.2. | Löwenheim's Theorem | 172 |
| 8.3. | Conclusions | 191 |
| 8.4. | Impact of Löwenheim's Paper | 195 |
| 9. | Skolem's Recasting | 197 |

Appendices

| | | |
|----|-------------------------|-----|
| 1. | Schröder's Lecture I | 207 |
| 2. | Schröder's Lecture II | 223 |
| 3. | Schröder's Lecture III | 251 |
| 4. | Schröder's Lecture V | 257 |
| 5. | Schröder's Lecture IX | 295 |
| 6. | Schröder's Lecture XI | 339 |
| 7. | Schröder's Lecture XII | 379 |
| 8. | Norbert Wiener's Thesis | 429 |
| | Bibliography | 445 |
| | Index | 461 |