

# Contents

---

Introduction	1
<b>1 Toral subalgebras in <math>p</math>-envelopes</b>	17
1.1 $p$ -envelopes	17
1.2 The absolute toral rank	23
1.3 Extended roots	30
1.4 Absolute toral ranks of parametrized families	39
1.5 Toral switching	46
<b>2 Lie algebras of special derivations</b>	58
2.1 Divided power mappings	59
2.2 Subalgebras defined by flags	73
2.3 Transitive embeddings of Lie algebras	79
2.4 Automorphisms and derivations	89
2.5 Filtrations and gradations	91
2.6 Minimal embeddings of filtered and associated graded Lie algebras	99
2.7 Miscellaneous	104
2.8 A universal embedding	111
2.9 The constructions can be made basis free	119
<b>3 Derivation simple algebras and modules</b>	133
3.1 Frobenius extensions	134
3.2 Induced modules	138
3.3 Block's theorems	151
3.4 Derivation semisimple associative algebras	163
3.5 Weisfeiler's theorems	167
3.6 Conjugacy classes of tori	176
<b>4 Simple Lie algebras</b>	180
4.1 Classical Lie algebras	180
4.2 Lie algebras of Cartan type	184
4.3 Melikian algebras	199
4.4 Simple Lie algebras in characteristic 3	209

<b>5 Recognition theorems</b>	217
5.1 Cohomology groups	217
5.2 From local to global Lie algebras	228
5.3 Representations	252
5.4 Generating Melikian algebras	258
5.5 The Weak Recognition Theorem	262
5.6 The Recognition Theorem	269
5.7 Wilson's Theorem	272
<b>6 The isomorphism problem</b>	283
6.1 A first attack	283
6.2 The compatibility property	295
6.3 Special algebras	299
6.4 Orbits of Hamiltonian forms	314
6.5 Hamiltonian algebras	329
6.6 Contact algebras	346
6.7 Melikian algebras	349
<b>7 Structure of simple Lie algebras</b>	357
7.1 Derivations	357
7.2 Restrictedness	363
7.3 Automorphisms	372
7.4 Gradings	386
7.5 Tori	388
7.6 $W(1; \underline{n})$	420
<b>8 Pairings of induced modules</b>	432
8.1 Cartan prolongation	432
8.2 Module pairings	449
8.3 Trigonalizability	461
<b>9 Toral rank 1 Lie algebras</b>	484
9.1 Solvable maximal subalgebras	484
9.2 Cartan subalgebras of toral rank 1	496
Notation	521
Bibliography	527
Index	539