

TABLE OF CONTENTS

INTRODUCTION	1
CHAPTER I: LINEAR DIFFERENTIAL EQUATIONS AND THE STURM-LIOUVILLE PROBLEM	9
§1. Differential equations and partial differential equations in the XVIII th century	9
§2. Fourier expansions	11
§3. The Sturm-Liouville theory	16
CHAPTER II: THE "CRYPTO-INTEGRAL" EQUATIONS	22
§1. The method of successive approximations	22
§2. Partial differential equations in the XIX th century	26
§3. The beginnings of potential theory	30
§4. The Dirichlet principle	35
§5. The Beer-Neumann method	39
CHAPTER III: THE EQUATION OF VIBRATING MEMBRANES	47
§1. H.A. Schwarz's 1885 paper	47
§2. The contributions of Poincaré	56
CHAPTER IV: THE IDEA OF INFINITE DIMENSION	71
§1. Linear algebra in the XIX th century	71
§2. Infinite determinants	75
§3. Groping towards function spaces	79
§4. The passage "from finiteness to infinity"	87
CHAPTER V: THE CRUCIAL YEARS AND THE DEFINITION OF HILBERT SPACE	97
§1. Fredholm's discovery	97
§2. The contributions of Hilbert	105
§3. The confluence of Geometry, Topology and Analysis	115
CHAPTER VI: DUALITY AND THE DEFINITION OF NORMED SPACES.	121
§1. The search for continuous linear functionals	121
§2. The L^p and l^p spaces	124
§3. The birth of normed spaces and the Hahn-Banach theorem	128
§4. The method of the gliding hump and Baire category	138

CHAPTER VII: SPECTRAL THEORY AFTER 1900	144
§1. F. Riesz's theory of compact operators	144
§2. The spectral theory of Hilbert	148
§3. The work of Weyl and Carleman	160
§4. The spectral theory of von Neumann	171
§5. Banach algebras	182
§6. Later developments	190
CHAPTER VIII: LOCALLY CONVEX SPACES AND THE THEORY OF DISTRIBUTIONS	210
§1. Weak convergence and weak topology	210
§2. Locally convex spaces	215
§3. The theory of distributions	221
CHAPTER IX: APPLICATIONS OF FUNCTIONAL ANALYSIS TO DIFFERENTIAL AND PARTIAL DIFFERENTIAL EQUATIONS	233
§1. Fixed point theorems	233
§2. Carleman operators and generalized eigenvectors	238
§3. Boundary problems for ordinary differential equations	243
§4. Sobolev spaces and <u>a priori</u> inequalities	248
§5. Elementary solutions, parametrices and pseudo-differential operators	252
REFERENCES	280
AUTHOR INDEX	299
SUBJECT INDEX	306