

---

---

# Contents

|   |     |
|---|-----|
| Acknowledgments   | ix  |
| Chapter 1. Introduction   | 1   |
| Bibliography  | 21  |
| Chapter 2. Arithmetic intersection theory on stacks   | 27  |
| 2.1 The one-dimensional case  | 27  |
| 2.2 $\text{Pic}(\mathcal{M})$ , $\widehat{\text{CH}}_{\mathbb{Z}}^1(\mathcal{M})$ , and $\widehat{\text{CH}}_{\mathbb{Z}}^2(\mathcal{M})$           | 30  |
| 2.3 Green functions   | 31  |
| 2.4 $\widehat{\text{Pic}}(\mathcal{M})$ , $\widehat{\text{CH}}_{\mathbb{Z}}^1(\mathcal{M})$ , and $\widehat{\text{CH}}_{\mathbb{Z}}^2(\mathcal{M})$ | 34  |
| 2.5 The pairing $\widehat{\text{CH}}^1(\mathcal{M}) \times \widehat{\text{CH}}^1(\mathcal{M}) \rightarrow \widehat{\text{CH}}^2(\mathcal{M})$       | 36  |
| 2.6 Arakelov heights  | 38  |
| 2.7 The arithmetic adjunction formula   | 39  |
| Bibliography  | 43  |
| Chapter 3. Cycles on Shimura curves   | 45  |
| 3.1 Shimura curves  | 46  |
| 3.2 Uniformization  | 47  |
| 3.3 The Hodge bundle  | 49  |
| 3.4 Special endomorphisms   | 51  |
| 3.5 Green functions   | 56  |
| 3.6 Special 0-cycles  | 57  |
| Bibliography  | 68  |
| Chapter 4. An arithmetic theta function   | 71  |
| 4.1 The structure of arithmetic Chow groups   | 71  |
| 4.2 The arithmetic theta function   | 77  |
| 4.3 The vertical component: definite theta functions  | 79  |
| 4.4 The analytic component: Maass forms   | 87  |
| 4.5 The Mordell-Weil component  | 94  |
| 4.6 Borcherds' generating function  | 96  |
| 4.7 An intertwining property  | 100 |
| Bibliography  | 102 |
| Chapter 5. The central derivative of a genus two Eisenstein series  | 105 |
| 5.1 Genus two Eisenstein series   | 105 |
| 5.2 Nonsingular Fourier coefficients  | 111 |

|   |   |            |
|---|---|------------|
| 5.3   | The Siegel-Weil formula   | 121        |
| 5.4   | Singular coefficients   | 137        |
| 5.5   | Eisenstein series of genus one  | 139        |
| 5.6   | $B_T$   | 140        |
| 5.7   | $W_T$   | 144        |
| 5.8   | The central derivative—the rank one case  | 154        |
| 5.9   | The constant term   | 161        |
|   | Bibliography  | 165        |
| <b>Chapter 6. The generating function for 0-cycles</b>              |   | <b>167</b> |
| 6.1   | The case $T > 0$ with $\text{Diff}(T, B) = \{p\}$ for $p \nmid D(B)$              | 169        |
| 6.2   | The case $T > 0$ with $\text{Diff}(T, B) = \{p\}$ for $p \mid D(B)$               | 172        |
| 6.3   | The case of nonsingular $T$ with $\text{sig}(T) = (1, 1)$ or $(0, 2)$             | 175        |
| 6.4   | Singular terms, $T$ of rank 1   | 177        |
| 6.5   | The constant term, $T = 0$  | 179        |
|   | Bibliography  | 180        |
| <b>Chapter 6 Appendix. The case <math>p = 2, p \mid D(B)</math></b> |   | <b>181</b> |
| 6A.1  | Statement of the result   | 181        |
| 6A.2  | Review of the special cycles $Z(j)$ , for $q(j) \in \mathbb{Z}_p \setminus \{0\}$ | 186        |
| 6A.3  | Configurations  | 188        |
| 6A.4  | Calculations  | 191        |
| 6A.5  | The first nondiagonal case  | 201        |
|   | Bibliography  | 204        |
| <b>Chapter 7. An inner product formula</b>                          |   | <b>205</b> |
| 7.1   | Statement of the main result  | 206        |
| 7.2   | The case $t_1 t_2$ is not a square  | 208        |
| 7.3   | A weakly admissible Green function  | 212        |
| 7.4   | A finer decomposition of special cycles   | 221        |
| 7.5   | Application of adjunction   | 225        |
| 7.6   | Contributions for $p \mid D(B)$   | 231        |
| 7.7   | Contributions for $p \nmid D(B)$  | 238        |
| 7.8   | Computation of the discriminant terms   | 245        |
| 7.9   | Comparison for the case $t_1, t_2 > 0$ , and $t_1 t_2 = m^2$                      | 252        |
| 7.10  | The case $t_1, t_2 < 0$ with $t_1 t_2 = m^2$                                      | 259        |
| 7.11  | The constant terms  | 262        |
|   | Bibliography  | 264        |
| <b>Chapter 8. On the doubling integral</b>                          |   | <b>265</b> |
| 8.1   | The global doubling integral  | 266        |
| 8.2   | Review of Waldspurger's theory  | 269        |
| 8.3   | An explicit doubling formula  | 279        |
| 8.4   | Local doubling integrals  | 285        |
| 8.5   | Appendix: Coordinates on metaplectic groups                                       | 320        |
|   | Bibliography  | 346        |

|  |            |
|--|------------|
| <b>Chapter 9. Central derivatives of L-functions</b> | <b>351</b> |
| 9.1 The arithmetic theta lift                        | 351        |
| 9.2 The arithmetic inner product formula             | 356        |
| 9.3 The relation with classical newforms             | 365        |
| Bibliography   | 369        |
| <b>Index</b>   | <b>371</b> |