## **CONTENTS**

**FOREWORD** 

PREFACE		vii
1. GROUP THEORY		
(a) GROUP AXIOMS AND EXAMPLES		
A Single Postulate for Groups	MICHAEL SLATER	1
The Existence of Free Groups	MICHAEL BARR	2
Non-abelian Groups of Order pq	S. K. Berberian	6
Bibliographic Entries: Group Axioms and Examples		9
(b) DECOMPOSITION OF GROUPS		
On Cancellation in Groups	R. Hirshon	9
Groups as Unions of Proper Subgroups	_	
	ND AZRIEL ROSENFELD	12
Bibliographic Entry: Decomposition of Groups		15
(c) HOMOMORPHISMS, AUTOMORPHISM GROUPS	MS, AND MAPPINGS	OF
Bibliographic Entries: Homomorphisms, Automorphis Mappings of Groups	ms, and	15
(d) ABELIAN GROUPS		
The Basis Theorem for Finitely Generated Abelian G	roups	
	Eugene Schenkman	16
A Proof of the Structure Theorem of Finite Abelian C	Froups E. R. GENTILE	17
Bibliographic Entries: Abelian Groups		19
(e) LINEAR GROUPS AND THE SYMME	TRIC GROUPS	
Note on the Alternating Group	E. L. Spitznagel, Jr.	19
An Historical Note on the Parity of Permutations	T. L. Bartlow	20
A New Method for the Determination of the Group of Isomorphisms of the Symmetric Group of Degree $n$	f H. A. Bender	23

x CONTENTS

The Group of the Pythagorean Numbers	J. Mariani	25
The Classical Groups as a Source of Algebraic Problems	C. W. Curtis	29
Bibliographic Entries: Linear Groups and the Symmetric	Groups	40
(f) SUBGROUPS AND ELEMENTS OF FINI	TE GROUPS	
Another Proof of Cauchy's Group Theorem	JAMES H. MCKAY	41
On a Theorem of Frobenius	RICHARD BRAUER	42
A Group-theoretic Proof of Wilson's Theorem	Walter Feit	46
Bibliographic Entries: Subgroups and Elements of Finite	Groups	46
(g) HISTORY		
The Foundation Period in the History of Group Theory		
J	OSEPHINE E. BURNS	46
On the History of Several Fundamental Theorems in Groups of Finite Order	the Theory of G. A. MILLER	55
Bibliographic Entry: History		58
(h) APPLICATIONS		
Campanological Groups	T. J. FLETCHER	58
The Group Generated by Central Symmetries, with A Polygons	applications to EDWARD KASNER	66
Bibliographic Entry: Applications		71
2. RING THEORY		
(a) EUCLIDEAN RINGS AND UNIQUE FACTORIZATION DOMAINS		
Unique Factorization of Gaussian Integers	Walter Rudin	72
A Characterization of Polynomial Domains over a Field		
	Tong-Shieng Rhai	73
A Note on Quadratic Euclidean Domains	P. J. Arpaia	76
Some Applications of a Morphism	RICHARD SINGER	78
A Principal Ideal Ring That Is Not a Euclidean Ring	JACK C. WILSON	79
Unique Factorization	PIERRE SAMUEL	82
Unique Factorization Domains	P. M. COHN	90
Bibliographic Entries: Euclidean Rings and Unique Domains	e Factorization	107

CONTENTS xi

(b) COMMUTATIVE RINGS		
Divisors of Zero in Polynomial Rings	W. R. Scott	108
Unimodular Complements	IRVING REINER	109
If $R[X]$ is Noetherian, $R$ Contains an Identity	ROBERT W. GILMER	110
An Existence Theorem for Non-Noetherian Rings	ROBERT W. GILMER	111
Bibliographic Entries: Commutative Rings		113
(c) MODULES		
Finitely Generated Modules over Principal Ideal Doma	ins W. J. Wong	113
Modules over Commutative Rings	W. G. LEAVITT	115
Note on Modules	Rolando E. Peinado	116
Bibliographic Entries: Modules		117
(d) DIVISION ALGEBRAS		
The Classification of Real Division Algebras	R. S. Palais	118
The Involutory Anti Automorphisms of the Quaternio		
	RABE VON RANDOW	120
Equations in Quaternions	Ivan Niven	121
Wedderburn's Theorem and a Theorem of Jacobson	I. N. Herstein	129
Another Proof of Wedderburn's Theorem	T. J. KACZYNSKI	131
Bibliographic Entries: Division Algebras		133
(e) NON-COMMUTATIVE RINGS		
The Algebra of Semi-Magic Squares	L. M. Weiner	133
A Relationship between Semi-Magic Squares and Pern	nutation Matrices	
	ARTHUR A. SAGLE	136
Annihilators in Polynomial Rings	NEAL H. McCoy	137
Orders for Finite Non Commutative Rings	D. B. ERICKSON	138
Inverses in Rings with Unity	C. W. BITZER	140
On the Matrix Equation $AB = I$	Paul Hill	140
On the Symmetry of Semi-Simple Rings A. Kert	ész and O. Steinfeld	142
A Theorem of Levitzki	Yuzo Utumi	144
Rings of Fractions	P. M. COHN	145
Nil Algebras and Periodic Groups IRWIN FISCHER	AND RUTH R. STRUIK	165
Bibliographic Entries: Non-Commutative Rings		177

xii CONTENTS

(f) NON-ASSOCIATIVE RINGS AND ALGE	BRAS	
Trace Functions on Algebras with Prime Characteristic		
	Hans Zassenhaus	177
On Associators in Jordan Algebras	C. E. Tsai	185
Linearization in Rings and Algebras		105
	and Seymour Kass	187
Bibliographic Entries: Non-Associative Rings and Algebra	ras	194
3. FIELD THEORY AND ALGEBRAIC NU	MBER THEORY	
(a) FIELD EXTENSIONS AND GALOIS TH	EORY	
On the Fundamental Theorem of Algebra	Hans Zassenhaus	195
A Note on the Algebraic Closure of a Field	ROBERT GILMER	207
Balanced Field Extensions	Joseph Lipman	209
On Extensions of Q by Square Roots	R. L. Rотн	211
On DeMoivre's Quintic	R. L. Borger	212
Bibliographic Entries: Field Extensions and Galois Theo	ry	215
(b) FINITE FIELDS		
Modular Fields	Saunders MacLane	216
A Combinatorial Proof of the Existence of Galois Fields	R. C. Mullin	231
The Number of Irreducible Polynomials of Degree $n$ ov	er GF(p)	
	G. J. SIMMONS	233
Bibliographic Entry: Finite Fields		235
(c) REDUCIBILITY OF POLYNOMIALS		
An Inductive Proof of Descartes' Rule of Signs	A. A. Albert	236
An Application of Determinants	Helen Skala	239
Bibliographic Entries: Reducibility of Polynomials		240
(d) VALUATION THEORY		
The p-adic Numbers of Hensel	C. C. MACDUFFEE	241
On Dividing a Square into Triangles	Paul Monsky	249
Bibliographic Entry: Valuation Theory		252
(e) ALGEBRAIC NUMBER THEORY		
A Number Field without a Relative Integral Basis ROBERT MACKENZIE AND	O JOHN SCHEUNEMAN	252

CON	NTENTS	xiii
Algebraic Number Fields and the Diophantine Equation $m^n = n^m$		
Algebraic Number Fields and the Diop	ALVIN HAUSNER	253
On the Prime Divisors of Polynomials		
•	IRVING GERST AND JOHN BRILLHART	259
What Is a Reciprocity Law?	B. F. Wyman	275
Density Questions in Algebraic Number	r Theory L. J. GOLDSTEIN	292
Bibliographic Entries: Algebraic Number	er Theory	301
4. LINEA	R ALGEBRA	
(a) EQUATIONS, DETERMI	NANTS, AND RELATED TOPICS	
On the Solution of a System of Linear I	Equations G. A. MILLER	302
On the Matrix Equation $BX = C$	H. T. Burgess	305
A Recurring Theorem on Determinants	Olga Taussky	308
On an Elementary Derivation of Cram		
	D. E. WHITFORD AND M. S. KLAMKIN	312
A Short Proof of Cramer's Rule	Stephen M. Robinson	313
A Note on the Sylvester-Franke Theore	em Harley Flanders	314
On the Generalized Inverse of an Arbi	trary Linear Transformation D. W. Robinson	317
Scalar Valued Mappings of Square Ma	trices S. CATER	321
A Proof of the Equality of Column and Row Rank of a Matrix		
, ,	Hans Liebeck	328
Bibliographic Entries: Equations, Determinants, and Related Topics		328
(b) CHARACTERISTIC POI	LYNOMIALS AND EIGENVALUES	
On Characteristic Roots of Matrix Prod	ducts W. M. Scott	329
The Matrices AB and BA	W. V. Parker	331
A Remark on Characteristic Polynomia	als Josef Schmid	332
A Note on Scalar Functions of Matrice	es RICHARD BELLMAN	333
A Note on Matrix Power Series	I. M. Sheffer	334
Bibliographic Entries: Characteristic Pe	olynomials and Eigenvalues	337
(c) INNER PRODUCTS AN	D QUADRATIC FORMS	
A Note on Normal Matrices	L. Mirsky	337
Topological Aspects of Sylvester's The Matrices	corem on the Inertia of Hermitian HANS SCHNEIDER	339

xiv CONTENTS

The Definition of a Quadratic Form	Andrew M. Gleason	344
A Simplified Proof of a Sufficient Condition for a Quadratic Form	Positive Definite S. M. SAMUELS	351
The Scarcity of Cross Products on Euclidean Spaces	BERTRAM WALSH	352
Reflections Have Reversed Vectors	A. M. Adelberg	359
Bibliographic Entries: Inner Products and Quadratic I	Forms	362
(d) CANONICAL FORMS		
A Condition for Diagonability of Matrices H. K. FA	arahat and L. Mirsky	362
On the Reduction of a Matrix to Diagonal Form  MARVIN EPSTEIN A	AND HARLEY FLANDERS	365
On the Minimization of Matrix Norms	L. Mirsky	368
Bibliographic Entries: Canonical Forms	2	370
(e) LINEAR ALGEBRA OVER FINITE FI	ELDS	
Regular Polygons over $GF[3^2]$	D. W. CROWE	370
Matrices over a Finite Field S. D. FISHER	AND M. N. ALEXANDER	374
Bibliographic Entry: Linear Algebra over Finite Field		376
(f) APPLICATIONS		
Linear Recurrence Relations	J. L. Brenner	377
On an Application of the Vandermonde Determinant	W. V. Parker	380
Integration by Matrix Inversion	WILLIAM SWARTZ	380
Variable Matrix Substitution in Algebraic Cryptograp	hy JACK LEVINE	382
The Search for Hadamard Matrices	*	201
SOLOMON W. GOLOMB AND		391
A Polygon Problem E. R. BERLEKAMP, E. N. GILI		397
Matrices in the Market Place	F. D. PARKER	405
Bibliographic Entries: Applications		409
(g) OTHER TOPICS		
Methods of Proof in Linear Algebra	HARLEY FLANDERS	410
Linear Operations on Matrices	Marvin Marcus	425
Commutativity in Finite Matrices	Olga Taussky	435
Bibliographic Entries: Other Topics		442

CONTENTS	X

535

## 5. HISTORY

Greek Methods of Solving Quadratic Equations	Walter C. Eells	444
Diophantus of Alexandria	J. D. Swift	455
Current Trends in Algebra	Garrett Birkhoff	463
Bibliographic Entries: History		
6. ADDITIONAL TOPIC	CS	
Sums of Squares	Olga Taussky	487
Formal Power Series	Ivan Niven	513
Bibliographic Entries: Additional Topics		532

**AUTHOR INDEX**