

# Contents

<b>Preface</b>	ix
<b>Introduction</b>	xii
<b>Chapter 1</b>	
A PRELIMINARY STUDY OF QUANTILE PROCESSES	
1.1. Empirical processes. Definitions	1
1.2. Quantile processes. Definitions	2
1.3. Quantile processes. Alternative definitions	3
1.4. Definition of, and some Glivenko–Cantelli theorems for, the normed sample quantile process	4
1.5. Preliminary notes on weak convergence of the normed sample quantile process. Brownian bridges	10
<b>Chapter 2</b>	
A WEAK CONVERGENCE OF THE NORMED SAMPLE QUANTILE PROCESS	15
<b>Chapter 3</b>	
STRONG APPROXIMATIONS OF THE NORMED QUANTILE PROCESS. THE KIEFER PROCESS	
3.1. Strong approximations of the uniform quantile process by a sequence of Brownian bridges and by a Kiefer process	17
3.2. The distance of the normed sample quantile process from the uniform quantile process	23
<b>Chapter 4</b>	
TWO APPROACHES TO CONSTRUCTING SIMULTANEOUS CONFIDENCE BOUNDS FOR QUANTILES	
4.1. Simultaneous confidence bounds for quantiles via density estimation	31
4.2. Direct simultaneous confidence bounds for the quantile function	35
<b>Chapter 5</b>	
WEAK CONVERGENCE OF QUANTILE PROCESSES IN WEIGHTED SUP-NORM METRICS AND FURTHER STRONG APPROXIMATIONS	
5.1. O'Reilly's weight function in the light of strong approximations, and weak convergence in such weighted sup-norm metrics	41

5.2. Weak convergence of the standardized normed sample quantile process (the case of the weight function $w(y) = (y(1-y))^{-1/2}$ ) . . . . .	53
5.3. An “up-to-date” summary of weak convergence of the normed sample quantile process $\rho_n$ in $\ \cdot/q\ $ -metrics . . . . .	55
5.4. An “up-to-date” summary of strong approximations of the normed sample quantile process . . . . .	63
<b>Chapter 6</b>	
ON BAHADUR’S REPRESENTATION OF SAMPLE QUANTILES AND ON KIEFER’S THEORY OF DEVIATIONS BETWEEN THE SAMPLE QUANTILE AND EMPIRICAL PROCESSES	
6.1. The Bahadur–Kiefer theory in terms of the distance of the sample quantile process from the uniform quantile process . . . . .	67
6.2. The distance of $R_n(y)$ from $R_n^*(y)$ . . . . .	72
<b>Chapter 7</b>	
QUADRATIC FORMS OF THE QUANTILE PROCESS, WEIGHTED SPACINGS AND TESTING FOR COMPOSITE GOODNESS-OF-FIT	
7.1. Parameters estimated quantile processes and some Cramér–von Mises-type functionals of them . . . . .	75
7.2. Empirical measures, ordered uniform spacings and testing for exponentiality . . . . .	81
7.3. On sums of weighted spacings for composite goodness-of-fit . . . . .	90
7.4. Further weak convergence for quantiles and spacings and on total time on test . . . . .	97
7.5. The Gini index and the Lorenz curve . . . . .	111
<b>Chapter 8</b>	
STRONG APPROXIMATIONS OF THE QUANTILE PROCESS OF THE PRODUCT-LIMIT ESTIMATOR	
8.1. Introduction . . . . .	115
8.2. Some small increments of a generalized Kiefer process and an LIL for the normalized generalized Kiefer process $K(s, t)/\Gamma^{1/2}(s, s)$ . . . . .	121
8.3. A strong approximation of the uniform PL-quantile process and some of its consequences . . . . .	127
8.4. On the distance of the uniform PL-quantile process from the PL-normed quantile process and consequences . . . . .	132
<b>Chapter 9</b>	
AN INVARIANCE PRINCIPLE FOR NEAREST NEIGHBOR EMPIRICAL DENSITY FUNCTIONS	
9.1. Introduction . . . . .	137
9.2. The conditions . . . . .	139
9.3. The main results . . . . .	139

<b>Chapter 10</b>	
A NEAREST-NEIGHBOR ESTIMATOR FOR THE SCORE	
FUNCTION	
10.1. Introduction . . . . .	145
10.2. On the consistency of a nearest-neighbor estimator for the score function . . . . .	146
<b>References</b> . . . . .	151