

Contents

| | |
|---|----|
| Introduction | v |
| Chapter 1 | |
| The History of the Soliton | 1 |
| 1a. John Scott Russell's discovery | 1 |
| 1b. Fermi–Pasta–Ulam | 3 |
| 1c. Kruskal, Zabusky and discovery of the soliton | 5 |
| 1d. The conservation laws and the Miura transformation | 9 |
| 1e. The inverse scattering transform | 11 |
| 1f. The Lax equation | 15 |
| 1g. Simultaneous developments in nonlinear optics and Bäcklund transformations | 16 |
| 1h. Soliton factories and later developments of the 1970's | 18 |
| 1i. Soliton miracles and the need for a unifying point of view | 19 |
| Chapter 2 | |
| Derivation of the Korteweg–deVries, Nonlinear Schrödinger and Other Important and Canonical Equations of Mathematical Physics | 23 |
| 2a. An outline of what we are going to do | 23 |
| 2b. Small amplitude, long waves in a channel of slowly changing depth. Equations of the KdV type | 25 |
| Exercises | 32 |
| 2c. The nonlinear Schrödinger and other envelope equations | 32 |
| Exercises | 38 |
| 2d. The Benjamin–Feir instability | 43 |
| Exercise | 47 |
| 2e. Whitham theory | 48 |
| Exercise | 57 |
| 2f. Other canonical equations | 57 |
| Chapter 3 | |
| Soliton Equation Families and Solution Methods | 61 |
| 3a. Introduction | 61 |
| 3b. The Korteweg–deVries equation family | 61 |
| Exercises | 65 |
| 3c. The AKNS hierarchy and its properties | 67 |
| Exercises | 70 |

| | |
|--|---------|
| 3d. The direct transform for the Schrödinger equation, or scattering on the infinite line | 72 |
| Exercises | 77 |
| 3e. The inverse transform | 78 |
| 3f. Time evolution of the scattering data and the effects of small variations in the potentials | 83 |
| 3g. Perturbation theory. Solitary waves in channels of slowly changing depth | 87 |
| Exercises | 97 |
| 3h. Multisoliton, rational and finite gap solutions | 98 |
| Chapter 4 | |
| The τ -Function, the Hirota Method, the Painlevé Property and Bäcklund Transformations for the Korteweg–deVries Family of Soliton Equations | 113 |
| 4a. Introduction | 113 |
| 4b. The τ -function | 113 |
| 4c. Symmetries, conservation laws and constants of the motion | 117 |
| 4d. The Hirota story | 120 |
| 4e. The Painlevé property | 129 |
| 4f. Bäcklund transformations | 134 |
| 4g. The appearance of a Kac–Moody algebra | 140 |
| Chapter 5 | |
| Connecting Links Among the Miracles of Soliton Mathematics | 145 |
| 5a. Overview | 145 |
| 5b. The Wahlquist–Estabrook approach | 150 |
| Exercise | 159 |
| 5c. Lax equations associated with $\widetilde{sl}(2, \mathbb{C})$ | 159 |
| Exercises | 165 |
| 5d. Conservation laws, fluxes, potentials and the Hirota equations | 169 |
| Exercise | 173 |
| 5e. The eigenvalue problem, asymptotic expansions and vertex operators | 173 |
| Exercise | 177 |
| 5f. Iso-spectral, iso-Riemann surfaces and iso-monodromic deformations | 181 |
| 5g. Gauge and Bäcklund transformations | 191 |
| 5h. The notion of grading | 206 |
| 5i. A second Hamiltonian structure | 209 |
| 5j. Inverse scattering and the Riemann–Hilbert problem, algebraic style | 210 |
| Exercises | 222 |
| 5k. The “sine-Gordon” flows | 223 |
| Exercises | 225 |
| 5l. The extension of $\widetilde{sl}(2, \mathbb{C})$ to $\hat{A}_1^{(1)}$ | 229 |
| References | 237 |