CONTENTS

CHAPTER V : *One-parameter groups* 7

§ 1. Subgroups and quotient groups of **R** 7

1. Closed subgroups of **R** 7
2. Quotient groups of **R** 7
3. Continuous homomorphisms of **R** into itself 9
4. Local definition of a continuous homomorphism of **R** into a topological group 9

§ 2. Measurement of magnitudes 12

§ 3. Topological characterization of the groups **R** and **T** 17

§ 4. Exponentials and logarithms 19

1. Definition of a^x and $\log_a x$ 19
2. Behaviour of the functions a^x and $\log_a x$ 21
3. Multipliable families of numbers > 0 22

Exercises for § 1 24
Exercises for § 2 25
Exercises for § 3 25
Exercises for § 4 28

Historical Note 29

CHAPTER VI. *Real number spaces and projective spaces* 31

§ 1. Real number space $\mathbf{R}^n$ 31

1. The topology of $\mathbf{R}^n$ 31
2. The additive group $\mathbf{R}^n$ 32

3. The vector space $\mathbf{R}^n$ 33
4. Affine linear varieties in $\mathbf{R}^n$ 34
5. Topology of vector spaces and algebras over the field $\mathbf{R}$ 36
6. Topology of matrix spaces over $\mathbf{R}$ 37

§ 2. Euclidean distance, balls and spheres 38
1. Euclidean distance in $\mathbf{R}^n$ 38
2. Displacements 39
3. Euclidean balls and spheres 40
4. Stereographic projection 42

§ 3. Real projective spaces 44
1. Topology of real projective spaces 45
2. Projective linear varieties 47
3. Embedding real number space in projective space 48
4. Application to the extension of real-valued functions 49
5. Spaces of projective linear varieties 50
6. Grassmannians 53

Exercises for § 1 55
Exercises for § 2 58
Exercises for § 3 61

Historical Note 64

CHAPTER VII. *The additive groups* $\mathbf{R}^n$ 67

§ 1. Subgroups and quotient groups of $\mathbf{R}^n$ 67
1. Discrete subgroups of $\mathbf{R}^n$ 68
2. Closed subgroups of $\mathbf{R}^n$ 71
3. Associated subgroups 73
4. Hausdorff quotient groups of $\mathbf{R}^n$ 76
5. Subgroups and quotient groups of $\mathbf{T}^n$ 77
6. Periodic functions 78

§ 2. Continuous homomorphisms of $\mathbf{R}^n$ and its quotient groups 79
1. Continuous homomorphisms of the group $\mathbf{R}^m$ into the group $\mathbf{R}^n$ 79
2. Local definition of a continuous homomorphisms of $\mathbf{R}^n$ into a topological group 79
3. Continuous homomorphisms of $\mathbf{R}^m$ into $\mathbf{T}^n$ 81
4. Automorphisms of $\mathbf{T}^n$ 82

§ 3. Infinite sums in the groups $\mathbf{R}^n$ 83
1. Summable families in $\mathbf{R}^n$ 83
2. Series in $\mathbf{R}^n$ 86

Exercises for § 1 87
Exercises for § 2 93
Exercises for § 3 95

Historical Note 97

CHAPTER VIII. *Complex numbers* 100

§ 1. Complex numbers, quaternions 100
1. Definition of complex numbers 100
2. The topology of C 102
3. The multiplicative group C* 103
4. The division ring of quaternions 104

§ 2. Angular measure, trigonometric functions 105
1. The multiplicative group U 105
2. Angles 107
3. Angular measure 108
4. Trigonometric functions 109
5. Angular sectors 112
6. Crosses 113

§ 3. Infinite sums and products of complex numbers 115
1. Infinite sums of complex numbers 115
2. Multipliable families in C* 115
3. Infinite products of complex numbers 117

§ 4. Complex number spaces and projective spaces 118
1. The vector space C^n 118
2. Topology of vector spaces and algebras over the field C 119
3. Complex projective spaces 119
4. Spaces of complex projective linear varieties 121

Exercises for § 1 123
Exercises for § 2 125
Exercises for § 3 126
Exercises for § 4 127

Historical Note 131

CHAPTER IX. *Use of real numbers in general topology* 137

§ 1. Generation of a uniformity by a family of pseudometrics; uniformizable spaces 137
1. Pseudometrics 137
2. Definition of a uniformity by means of a family of pseudometrics 138

3. Properties of uniformities defined by families of pseudometrics 141
4. Construction of a family of pseudometrics defining a uniformity 142
5. Uniformizable spaces 144
6. Semi-continuous functions on a uniformizable space ... 146

§ 2. Metric spaces and metrizable spaces 147
1. Metrics and metric spaces 147
2. Structure of a metric space 148
3. Oscillation of a function 151
4. Metrizable uniform spaces 151
5. Metrizable topological spaces 152
6. Use of countable sequences 153
7. Semi-continuous functions on a metrizable space 155
8. Metrizable spaces of countable type 155
9. Compact metric spaces; compact metrizable spaces ... 157
10. Quotient spaces of metrizable spaces 159

§ 3. Metrizable groups, valued fields, normed spaces and algebras 161
1. Metrizable topological groups 161
2. Valued division rings 165
3. Normed spaces over a valued division ring 169
4. Quotient spaces and product spaces of normed spaces . 172
5. Continuous multilinear functions 173
6. Absolutely summable families in a normed space 174
7. Normed algebras over a valued field 175

§ 4. Normal spaces 179
1. Definition of normal spaces 179
2. Extension of a continuous real-valued function 182
3. Locally finite open coverings of a closed set in a normal space 185
4. Paracompact spaces 187
5. Paracompactness of metrizable spaces 188

§ 5. Baire spaces 190
1. Nowhere dense sets 190
2. Meagre sets 192
3. Baire spaces 192
4. Semi-continuous functions on a Baire space 194

§ 6. Polish spaces, Souslin spaces, Borel sets 195
1. Polish spaces 195
2. Souslin spaces 197

3. Borel sets 199
4. Zero-dimensional spaces and Lusin spaces 200
5. Sieves 202
6. Separation of Souslin sets 204
7. Lusin spaces and Borel sets 205
8. Borel sections 206
9. Capacitability of Souslin sets 208

Appendix : Infinite products in normed algebras 211
1. Multipliable sequences in a normed algebra 211
2. Multipliability criteria 212
3. Infinite products 215

Exercises for § 1 218
Exercises for § 2 226
Exercises for § 3 236
Exercises for § 4 239
Exercises for § 5 250
Exercises for § 6 258
Exercises for the Appendix 268

Historical Note 271

CHAPTER X. *Function spaces* 274

§ 1. The uniformity of $\mathfrak{S}$-convergence 274
1. The uniformity of uniform convergence 274
2. $\mathfrak{S}$-convergence 275
3. Examples of $\mathfrak{S}$-convergence 277
4. Properties of the spaces $\mathcal{F}_{\mathfrak{S}}(X; Y)$ 278
5. Complete subsets of $\mathcal{F}_{\mathfrak{S}}(X; Y)$ 279
6. $\mathfrak{S}$-convergence in spaces of continuous mappings 280

§ 2. Equicontinuous sets 283
1. Definition and general criteria 283
2. Special criteria for equicontinuity 287
3. Closure of an equicontinuous set 289
4. Pointwise convergence and compact convergence on equicontinuous sets 289
5. Compact sets of continuous mappings 290

§ 3. Special function spaces 293
1. Spaces of mappings into a metric space 293
2. Spaces of mappings into a normed space 295
3. Countability properties of spaces of continuous functions 298

4. The compact-open topology 300
5. Topologies on groups of homeomorphisms 305

§ 4. Approximation of continuous real-valued functions 308

1. Approximation of continuous functions by functions belonging to a lattice 308
2. Approximation of continuous functions by polynomials 311
3. Application: approximation of continuous real-valued functions defined on a product of compact spaces 314
4. Approximation of continuous mappings of a compact space into a normed space 314

Exercises for § 1 318
Exercises for § 2 321
Exercises for § 3 327
Exercises for § 4 337

Historical Note 347

INDEX OF NOTATION 349

INDEX OF TERMINOLOGY 351