

# CONTENTS

CHAPTER 1. WHY MATHEMATICS? . . . . .	1
CHAPTER 2. THE COURSE OF MATHEMATICS: AN HISTORICAL ORIENTATION . . . . .	11
2-1 Introduction . . . . .	11
2-2 Mathematics in early civilizations . . . . .	11
2-3 The classical Greek period . . . . .	14
2-4 The Alexandrian Greek period . . . . .	17
2-5 The Hindus and Arabs . . . . .	18
2-6 Early and medieval Europe . . . . .	19
2-7 The Renaissance . . . . .	20
2-8 Developments from 1550 to 1800 . . . . .	21
2-9 Developments from 1800 to the present . . . . .	24
2-10 The human aspect of mathematics . . . . .	26
CHAPTER 3. THE WAYS OF MATHEMATICS . . . . .	29
3-1 Introduction . . . . .	29
3-2 The concepts of mathematics . . . . .	29
3-3 Idealization . . . . .	37
3-4 Methods of reasoning . . . . .	38
3-5 Mathematical proof . . . . .	41
3-6 Axioms and definitions . . . . .	47
3-7 The creation of mathematics . . . . .	48
3-8 Summary . . . . .	52
CHAPTER 4. NUMBER: THE FUNDAMENTAL CONCEPT . . . . .	54
4-1 Introduction . . . . .	54
4-2 Whole numbers and fractions . . . . .	54
4-3 Irrational numbers . . . . .	59
4-4 Negative numbers . . . . .	67
4-5 The axioms concerning numbers . . . . .	69
4-6 Applications of the number system . . . . .	72
CHAPTER 5. ALGEBRA, THE HIGHER ARITHMETIC . . . . .	82
5-1 Introduction . . . . .	82
5-2 The language of algebra . . . . .	82
5-3 Algebraic transformations . . . . .	85
5-4 Equations involving unknowns . . . . .	88
5-5 The general second-degree equation . . . . .	94
5-6 The history of equations of higher degree . . . . .	100

**CHAPTER 6. THE NATURE AND USES OF EUCLIDEAN GEOMETRY . . . . . 103**

6-1 The beginnings of geometry . . . . . 103

6-2 The content of Euclidean geometry . . . . . 105

6-3 Some mundane uses of Euclidean geometry . . . . . 111

6-4 Euclidean geometry and the study of light . . . . . 117

6-5 Conic sections . . . . . 123

6-6 The conic sections and light . . . . . 125

6-7 The cultural influence of Euclidean geometry . . . . . 129

6-8 Euclidean geometry within the Greek cultural world . . . . . 131

**CHAPTER 7. CHARTING THE EARTH AND THE HEAVENS . . . . . 136**

7-1 The Alexandrian world . . . . . 136

7-2 Basic concepts of trigonometry . . . . . 141

7-3 Some mundane uses of trigonometric ratios . . . . . 145

7-4 Charting the earth . . . . . 148

7-5 Charting the heavens . . . . . 154

7-6 Further progress in the study of light . . . . . 158

**CHAPTER 8. THE MATHEMATICAL ORDER OF NATURE . . . . . 169**

8-1 The Greek concept of nature . . . . . 169

8-2 Pre-Greek and early Greek views of nature . . . . . 169

8-3 The Pythagorean and Platonic views of nature . . . . . 172

8-4 Greek astronomical theories . . . . . 174

8-5 The evidence for the mathematical design of nature . . . . . 177

8-6 The destruction of the Greek world . . . . . 180

**CHAPTER 9. THE AWAKENING OF EUROPE . . . . . 183**

9-1 The medieval civilization of Europe . . . . . 183

9-2 Mathematics in the medieval period . . . . . 186

9-3 First revival of the Greek works . . . . . 189

9-4 Revolutionary influence in Europe . . . . . 192

9-5 Intellectual revolt . . . . . 195

9-6 New doctrines of the Renaissance . . . . . 197

9-7 The religious motivation in the study of nature . . . . . 200

**CHAPTER 10. MATHEMATICS AND PAINTING IN THE RENAISSANCE . . . . . 203**

10-1 Introduction . . . . . 203

10-2 Gropings toward a scientific system of perspective . . . . . 203

10-3 Realism leads to mathematics . . . . . 209

10-4 The basic idea of mathematical perspective . . . . . 212

10-5 Some mathematical theorems on perspective drawing . . . . . 215

10-6 Renaissance paintings employing mathematical perspective . . . . . 221

10-7 Other values of mathematical perspective . . . . . 229

CHAPTER 11. PROJECTIVE GEOMETRY . . . . .	232
11-1 The problem suggested by projection and section . . . . .	232
11-2 The work of Desargues . . . . .	234
11-3 The work of Pascal . . . . .	239
11-4 The principle of duality . . . . .	242
11-5 The relationship between projective and Euclidean geometries . . . . .	247
CHAPTER 12. THE REVOLUTION IN ASTRONOMY . . . . .	250
12-1 Introduction . . . . .	250
12-2 The work of Copernicus . . . . .	250
12-3 The work of Kepler . . . . .	253
12-4 The objections to a heliocentric theory . . . . .	258
12-5 The arguments for the heliocentric theory . . . . .	261
12-6 The scientific import of the heliocentric theory . . . . .	264
12-7 The cultural influences of the heliocentric theory . . . . .	265
CHAPTER 13. COORDINATE GEOMETRY . . . . .	271
13-1 Descartes and Fermat . . . . .	271
13-2 The need for new methods in geometry . . . . .	274
13-3 The concepts of equation and curve . . . . .	276
13-4 The parabola . . . . .	284
13-5 Finding a curve from its equation . . . . .	289
13-6 The ellipse . . . . .	291
13-7 The equations of surfaces . . . . .	293
13-8 Four-dimensional geometry . . . . .	296
13-9 Summary . . . . .	297
CHAPTER 14. THE MATHEMATIZATION OF SCIENCE . . . . .	300
14-1 Mathematics and modern science . . . . .	300
14-2 The growth of modern science . . . . .	300
14-3 The search for scientific method . . . . .	306
14-4 The scientific method of Galileo . . . . .	308
CHAPTER 15. THE SIMPLEST FORMULAS IN ACTION . . . . .	318
15-1 Introduction . . . . .	318
15-2 Functions and formulas . . . . .	318
15-3 The formulas describing the motion of dropped objects . . . . .	321
15-4 The formulas describing the motion of objects thrown downward . . . . .	327
15-5 Formulas for the motion of bodies projected upward . . . . .	328
15-6 Mass and weight . . . . .	331
15-7 Vertical motion in water . . . . .	333
15-8 Motion along an inclined plane . . . . .	335
15-9 Motion along planes with different slopes . . . . .	341
15-10 Summary . . . . .	344

CHAPTER 16. PARAMETRIC EQUATIONS AND CURVILINEAR MOTION . . . . .	346
16-1 Introduction . . . . .	346
16-2 The concept of parametric equations . . . . .	347
16-3 The motion of a projectile dropped from an airplane . . . . .	348
16-4 The motion of projectiles launched by cannons . . . . .	352
16-5 The motion of projectiles fired at an arbitrary angle . . . . .	357
16-6 Summary . . . . .	363
CHAPTER 17. THE APPLICATION OF FORMULAS TO GRAVITATION . . . . .	365
17-1 Introduction . . . . .	365
17-2 The problem of relating earthly and heavenly motions . . . . .	365
17-3 A sketch of Newton's life . . . . .	367
17-4 The law of gravitation and the second law of motion . . . . .	369
17-5 Further discussion of weight and mass . . . . .	375
17-6 Some deductions from the law of gravitation . . . . .	377
17-7 The rotation of the earth . . . . .	383
17-8 Gravitation and the Keplerian laws . . . . .	386
17-9 Implications of the theory of gravitation . . . . .	390
CHAPTER 18. THE DIFFERENTIAL CALCULUS . . . . .	396
18-1 Introduction . . . . .	396
18-2 The problems leading to the calculus . . . . .	396
18-3 The concept of instantaneous rate of change . . . . .	398
18-4 The concept of instantaneous speed . . . . .	400
18-5 The method of increments . . . . .	402
18-6 The method of increments applied to general functions . . . . .	405
18-7 The geometrical meaning of the derivative . . . . .	410
18-8 The maximum and minimum values of functions . . . . .	413
CHAPTER 19. THE INTEGRAL CALCULUS . . . . .	417
19-1 Differential and integral calculus compared . . . . .	417
19-2 Finding the formula from the given rate of change . . . . .	417
19-3 Applications to problems of motion . . . . .	418
19-4 Areas obtained by integration . . . . .	422
19-5 The calculation of work . . . . .	426
19-6 The calculation of escape velocity . . . . .	429
19-7 The equation of the cable of a bridge . . . . .	431
19-8 The concept of a differential equation . . . . .	432
19-9 The integral as the limit of a sum . . . . .	434
19-10 Some relevant history of the limit concept . . . . .	439
CHAPTER 20. THE AGE OF REASON . . . . .	443
20-1 Introduction . . . . .	443
20-2 Materialism . . . . .	444

20-3	Mechanism . . . . .	446
20-4	Determinism . . . . .	448
20-5	Theories of knowledge . . . . .	449
20-6	Psychology . . . . .	455
20-7	Biology . . . . .	457
20-8	Respite . . . . .	459
CHAPTER 21. RELIGION IN THE AGE OF REASON . . . . .		461
21-1	Introduction . . . . .	461
21-2	The grounds of the conflict . . . . .	461
21-3	The beliefs of the creators of the Age of Reason . . . . .	463
21-4	Rational movements in religion . . . . .	466
21-5	The decline of superstition . . . . .	471
21-6	The rise of toleration . . . . .	472
21-7	The problem of ethics . . . . .	474
CHAPTER 22. REASON IN LITERATURE AND AESTHETICS . . . . .		479
22-1	Introduction . . . . .	479
22-2	The language of reason . . . . .	479
22-3	The reform of style and spirit . . . . .	481
22-4	The age of prose . . . . .	485
22-5	The content of the literature . . . . .	486
22-6	Aesthetics . . . . .	487
22-7	The revolt against reason . . . . .	489
22-8	Retrospect . . . . .	491
CHAPTER 23. TRIGONOMETRIC FUNCTIONS AND OSCILLATORY MOTION . . . . .		493
23-1	Introduction . . . . .	493
23-2	The motion of a bob on a spring . . . . .	494
23-3	The sinusoidal functions . . . . .	495
23-4	The mathematical analysis of the motion of the bob . . . . .	503
23-5	Summary . . . . .	509
CHAPTER 24. THE TRIGONOMETRIC ANALYSIS OF MUSICAL SOUNDS . . . . .		510
24-1	Introduction . . . . .	510
24-2	The nature of simple sounds . . . . .	511
24-3	The method of addition of ordinates . . . . .	516
24-4	The analysis of complex sounds . . . . .	518
24-5	Subjective properties of musical sounds . . . . .	522
24-6	Some practical applications of the mathematical analysis . . . . .	524
24-7	Applications to physiology . . . . .	525
24-8	Summary . . . . .	527

CHAPTER 25. TRIGONOMETRIC FUNCTIONS AND ELECTROMAGNETISM . . . . .	529
25-1 Introduction . . . . .	529
25-2 Historical background . . . . .	529
25-3 The generation of alternating current . . . . .	531
25-4 Electromagnetic waves . . . . .	538
25-5 Electromagnetic waves and light . . . . .	544
25-6 The range of electromagnetic waves . . . . .	546
25-7 Electromagnetic waves and the sense of sight . . . . .	546
25-8 Electromagnetic theory and the physical world . . . . .	548
CHAPTER 26. NON-EUCLIDEAN GEOMETRIES AND THEIR SIGNIFICANCE . . . . .	553
26-1 Introduction . . . . .	553
26-2 The historical background . . . . .	553
26-3 The mathematical content of Gauss's non-Euclidean geometry . . . . .	560
26-4 Riemann's non-Euclidean geometry . . . . .	562
26-5 The applicability of non-Euclidean geometry . . . . .	563
26-6 The applicability of non-Euclidean geometry under a new interpretation of line . . . . .	566
26-7 Non-Euclidean geometry and the nature of mathematics . . . . .	572
26-8 The implications of non-Euclidean geometry for other branches of our culture . . . . .	575
CHAPTER 27. ARITHMETICS AND THEIR ALGEBRAS . . . . .	579
27-1 Introduction . . . . .	579
27-2 The applicability of the real number system . . . . .	579
27-3 Modular arithmetics and their algebras . . . . .	582
27-4 The algebra of sets . . . . .	587
27-5 Arithmetics and algebras as structures . . . . .	592
27-6 Mathematics and models . . . . .	593
CHAPTER 28. THE DEDUCTIVE APPROACH TO THE SOCIAL SCIENCES . . . . .	595
28-1 Introduction . . . . .	595
28-2 The rational reconstruction of political science . . . . .	595
28-3 The philosophy of utilitarianism . . . . .	599
28-4 The rational approach to economics . . . . .	603
28-5 The reform of the philosophy of history . . . . .	608
28-6 The accomplishment in the social sciences . . . . .	610
CHAPTER 29. THE STATISTICAL APPROACH TO THE SOCIAL AND BIOLOGICAL SCIENCES . . . . .	613
29-1 Introduction . . . . .	613
29-2 A brief historical review . . . . .	614
29-3 Averages . . . . .	615
29-4 Dispersion . . . . .	617

29-5	The graph and the normal curve . . . . .	619
29-6	Fitting a formula to data . . . . .	625
29-7	Correlation . . . . .	630
29-8	Cautions concerning the uses of statistics . . . . .	632
CHAPTER 30. THE THEORY OF PROBABILITY . . . . .		636
30-1	Introduction . . . . .	636
30-2	Probability for equally likely outcomes . . . . .	637
30-3	Probability as relative frequency . . . . .	642
30-4	Probability in continuous variation . . . . .	643
30-5	Binomial distributions . . . . .	646
30-6	The problems of sampling . . . . .	651
30-7	Probability in the physical sciences . . . . .	652
30-8	The statistical view of nature . . . . .	653
CHAPTER 31. THE NATURE AND VALUES OF MATHEMATICS . . . . .		660
31-1	Introduction . . . . .	660
31-2	The structure of mathematics . . . . .	660
31-3	The values of mathematics for the study of nature . . . . .	666
31-4	The aesthetic and intellectual values . . . . .	669
31-5	Mathematics and rationalism . . . . .	673
31-6	The limitations of mathematics . . . . .	674
TABLE OF TRIGONOMETRIC RATIOS . . . . .		679
ANSWERS TO SELECTED EXERCISES . . . . .		683
INDEX . . . . .		691