

Contents

Preface	ix
Acknowledgments	xii
Introduction	1
<i>A Non-Technical Introduction to Ethnomethodological Investigations of the Foundations of Mathematics through the Use of a Theorem of Euclidean Geometry</i>	1
<i>A Guide to the Reading of this Book</i>	15
Part I	21
Introduction. The Phenomenon: The Existence of Classical Studies of Mathematicians' Work	23
1 A Review of the Classical Representation of Mathematicians' Work as Formal Logistic Systems	25
2 An Introduction to Gödel's Incompleteness Theorems: Their Metamathematical Interpretation Contrasted with the Proposal to Study Their Natural Accountability in and as the Lived-Work of Their Proofs	31
Part II A Descriptive Analysis of the Work of Proving Gödel's First Incompleteness Theorem	37
3 Gödel Numbering and Related Topics: Background Materials for a Proof of Gödel's Theorem	39

CONTENTS

4	The Double-Diagonalization/'Proof': Features of the Closing Argument of a Proof of Gödel's Theorem as Lived-Work	45
5	A Technical Lemma: A Lemma Used in the Proof of Gödel's Theorem; Its Origins as a Technical Residue of the Work of Proving Gödel's Theorem within that Self-Same Work	51
6	Primitive Recursive Functions and Relations: An Initial Discussion of the Irremediable Connection between a Prover's Use of the Abbreviatory Practices/Practical Techniques of Working with Primitive Recursive Functions and Relations and the Natural Accountability of a Proof of Gödel's Theorem	57
7	A Schedule of Proofs: An Extended Analysis of the Lived-Work of Producing the Body of a Proof of Gödel's Theorem	65
	<i>A A Schedule of Proofs</i>	65
	<i>B A Schedule of Proofs as Lived-Work</i>	69
	1 As it is used in developing a schedule of proofs, a Gödel numbering is not an abstractly defined correspondence between the symbols, expressions, and sequences of expressions of P and the natural numbers; it is a technique of proving	69
	2 The schedule of proofs has a 'directed' character: it leads to and is organized so as to permit, the construction of G as a primitive recursive relation	76
	3 The selection and arrangement of these-particular propositions as composing this-particular, intrinsically sequentialized order of proving is the situated achievement of the work of producing that schedule of proofs	80
	(a) Six themes concerning the lived-work of producing a schedule of proofs	81
	(b) The construction of a schedule of proofs so as to provide an apparatus within itself for the analysis of the work of its own construction	100
	(c) The correspondence between the propositions of a schedule of proofs and the syntax of formal number theory as an achievement of the schedule of proofs itself	117
	(d) A review of the work of generalizing a schedule of proofs so as to elucidate the character of the development and organization of a schedule as a radical problem, for the prover, in the production of social order	125

4	The work of providing a consistent notation for a schedule of proofs articulates that schedule as one coherent object	137
8	A Structure of Proving	149
	<i>A The Characterization Problem: The Problem of Specifying What Identifies a Proof of Gödel's Theorem as a Naturally Accountable Proof of Just That Theorem; The Texture of the Characterization Problem and the Constraints on Its Solution; The Characterization Problem as the Foundational Problem</i>	149
	<i>B Generalizing the Proof of Gödel's Theorem (As a Means of Gaining Technical Access to the Characterization Problem)</i>	154
	<i>C A Structure of Proving: The Availability to a Prover of the Proof of Gödel's Theorem as a Structure of Practices; The Proof as the Pair The-Proof/The-Practices-of-Proving-to-Which-That-Proof-is-Irremediably-Tied</i>	169
	Part III Conclusion	173
9	Summary and Directions for Further Study	175
	<i>A Classical Studies of Mathematical Practice: A Review of the Book's Argument</i>	175
	<i>B Prospectus: Mathematicians' Work as Structure Building</i>	177
	Appendix	179
	The Use of Ethnomethodological Investigations of Mathematicians' Work for Reformulating the Problem of the Relationship between Mathematics and Theoretical Physics as a Real-World Researchable Problem in the Production of Social Order	181
	Notes	190
	Bibliography	237