

Contents

Preface	xi
Motivation	xi
Epitome	xii
Acknowledgments	xii
I Fundamentals of Turbulence	xv
1 Introduction	1
1.1 The Turbulence Problem	1
1.2 Closure Modeling	6
1.3 Categories of Turbulent Flow	8
2 Mathematical and Statistical Background	13
2.1 Dimensional Analysis	13
2.1.1 Scales of Turbulence	16
2.2 Statistical Tools	17
2.2.1 Averages and P.D.F.'s	17
2.2.2 Correlations	23
2.3 Cartesian Tensors	30
2.3.1 Isotropic Tensors	32
2.3.2 Tensor Functions of Tensors; Cayley-Hamilton Theorem	33
2.4 Transformation to Curvilinear Coordinates	38
2.4.1 Covariant and Contravariant Tensor Quantities	38
2.4.2 Differentiation of Tensors	40
2.4.3 Physical Components	42
3 Reynolds Averaged Navier-Stokes Equations	47
3.1 Reynolds Averaged Equations	49
3.2 The Terms of the Kinetic Energy and Reynolds Stress Budgets	51
3.3 Passive Contaminant Transport	55
4 Parallel and Self-Similar Shear Flows	59
4.1 Plane Channel Flow	59
4.1.1 The Logarithmic Layer	62
4.1.2 Roughness	65

4.2	The Boundary Layer	66
4.2.1	Entrainment	70
4.3	Free Shear Layers	71
4.3.1	Spreading Rates	76
4.3.2	Remarks on Self-Similar Boundary Layers	77
4.4	Heat and Mass Transfer	78
4.4.1	Parallel Flow and Boundary Layers	78
4.4.2	Dispersion from Elevated Sources	82
5	Vorticity and Vortical Structures	89
5.1	Structures	90
5.1.1	Free Shear Layers	91
5.1.2	Boundary Layers	95
5.1.3	Non-Random Vortices	99
5.2	Vorticity and Dissipation	99
5.2.1	Vortex Stretching and Relative Dispersion	102
5.2.2	The Mean-Squared Vorticity Equation	103
II	Single Point Closure Modeling	107
6	Models with Scalar Variables	109
6.1	Boundary Layer Methods	110
6.1.1	Integral Boundary Layer Methods	111
6.1.2	The Mixing Length Model	114
6.2	The $k - \varepsilon$ Model	118
6.2.1	Analytical Solutions to the $k - \varepsilon$ Model	120
6.2.2	Boundary Conditions and Near-wall Modifications	124
6.2.3	Weak Solution at Edges of Free-Shear Flow; Free-Stream Sensitivity	131
6.3	The $k - \omega$ Model	132
6.4	The Stagnation Point Anomaly	136
6.5	The Question of Transition	138
6.6	Eddy Viscosity Transport Models	140
7	Models with Tensor Variables	147
7.1	Second Moment Transport	147
7.1.1	A Simple Illustration	148
7.1.2	Closing the Reynolds Stress Transport Equation	148
7.1.3	Models for the Slow Part	150
7.1.4	Models for the Rapid Part	153
7.2	Analytic Solutions to SMC Models	158
7.2.1	Homogeneous Shear Flow	160
7.2.2	Curved Shear Flow	162
7.3	Non-homogeneity	166
7.3.1	Turbulent Transport	167
7.3.2	Near-Wall Modeling	168
7.3.3	No-Slip	169
7.3.4	Non-Local Wall Effects	170

7.4	Reynolds Averaged Computation	181
7.4.1	Numerical Issues	181
7.4.2	Examples of Reynolds Averaged Computation	185
8	Advanced Topics	201
8.1	Further Modeling Principles	201
8.1.1	Galilean Invariance and Frame Rotation	202
8.1.2	Realizability	205
8.2	Moving Equilibrium Solutions of SMC	207
8.2.1	Criterion for Steady Mean Flow	208
8.2.2	Solution in Two-Dimensional Mean Flow	209
8.2.3	Bifurcations	212
8.3	Passive Scalar Flux Modeling	215
8.3.1	Scalar Diffusivity Models	215
8.3.2	Tensor Diffusivity Models	216
8.3.3	Scalar Flux Transport	218
8.3.4	Scalar Variance	221
8.4	Active Scalar Flux Modeling: Effects of Buoyancy	222
8.4.1	Second Moment Transport Models	224
8.4.2	Stratified Shear Flow	226
III	Theory of Homogeneous Turbulence	229
9	Mathematical Representations	231
9.1	Fourier Transforms	232
9.2	The 3-D Energy Spectrum of Homogeneous Turbulence	233
9.2.1	The Spectrum Tensor and Velocity Covariances	234
9.2.2	Modeling the Energy Spectrum	236
10	Navier-Stokes Equations in Spectral Space	247
10.1	Convolution Integrals as Triad Interaction	247
10.2	Evolution of Spectra	249
10.2.1	Small k -Behavior and Energy Decay	249
10.2.2	The Energy Cascade	250
10.2.3	Final Period of Decay	254
11	Rapid Distortion Theory	257
11.1	Irrotational Mean Flow	258
11.1.1	Cauchy Form of the Vorticity Equation	258
11.1.2	Distortion of a Fourier Mode	261
11.1.3	Calculation of Covariances	262
11.2	General Homogeneous Distortions	267
11.2.1	Homogeneous Shear	268
11.2.2	Turbulence Near a Wall	271
References	277
Index	283