Contents

Author's Preface vii

Translator's Foreword ix

Chapter 1. Basic Concepts 1
1.1. Measure Theoretic Probability (1) Intuitive Background 1
1.2. Probability Distribution 3
1.3. Measure Theoretic Probability (2) Mathematical Structure 6
1.4. Distribution Function, Characteristic Function, Mean, Variance 8
1.5. Stochastic Process 13

Chapter 2. Additive Processes 15
2.1. Definition of Additive Process 15
2.2. Examples of Additive Processes 16
2.3. Inequalities Concerning Sums of Independent Random Variables 17
2.4. 0-1 Law 19
2.5. Convergence of Additive Sequences 21
2.6. Dispersion 24
2.7. Simple Properties of Additive Processes 28
2.8. Separability of Stochastic Processes 32
2.9. Separable Poisson Processes 33
2.10. Separable Wiener Processes 36
2.11. Additive Processes Continuous in Probability and Infinitely Divisible Distributions 39
2.12. Structure of Separable Additive Processes Continuous in Probability 43
2.13. Canonical Form of Infinitely Divisible Distributions 44
2.14. Various Methods for Construction of Poisson Processes 47
2.15. Compound Poisson Processes 49
2.16. Stable Distributions and Stable Processes 50

Chapter 3. Stationary Processes 57
3.1. Definition of Stationary Process 57
3.2. Preliminary Material Related to Investigations of Stationary Processes 58
3.3. Spectral Decomposition of Weakly Stationary Processes 60
3.4. Spectral Decomposition of Sample Processes of Weakly Stationary Processes 62
3.5. Ergodic Theorem Concerning Strongly Stationary Processes 64
3.6. Complex Normal System 67
3.7. Normal Stationary Processes 71
3.8. Wiener Integrals and Multiple Wiener Integrals 73

3.9. Ergodicity of Normal Stationary Processes 74
3.10. Generalizations of Stationary Processes 77

Chapter 4. Markov Processes 85
4.1. Conditional Probability 85
4.2. Conditional Expectation 86
4.3. Martingales 88
4.4. Transition Probabilities 88
4.5. Semi-Groups and Dual Semi-Groups Associated with Transition Probabilities 90
4.6. Hille-Yosida Theory (i) 92
4.7. Hille-Yosida Theory (ii). Construction of Semi-Group 95
4.8. Generators of Transition Probabilities (i). General Theory 98
4.9. Generators of Transition Probabilities (ii). Examples 101
4.10. Markov Processes (i). Markov Property 104
4.11. Markov Processes (ii). Properties of Sample Processes 106
4.12. Markov Processes (iii). Strong Markov Property 108
4.13. Markov Times 112
4.14. Dynkin's Theorem on Generators 115
4.15. Examples of Markov Processes 117
4.16. Temporally Homogeneous Additive Processes 120
4.17. Birth and Death Processes 121

Chapter 5. Diffusion 127
5.1. Diffusive Points 127
5.2. Ray's Theorem 127
5.3. Local Generators 130
5.4. Classification of One-Dimensional Diffusive Points 132
5.5. Feller's Canonical Scale 134
5.6. Feller's Canonical Measure 138
5.7. Feller's Canonical Form 139
5.8. Local Generators at Generalized Shunts 143
5.9. Distribution of the First Passage Time 145
5.10. Classical Diffusion Processes 148
5.11. Classification of Boundary Points with Respect to Feller's Operator $D_m D_s^+$ 151
5.12. Particular Solutions of the Homogeneous Equation $(\lambda - D_m D_s^+)u = 0$ $(\lambda > 0)$ 153
5.13. General Solutions of the Homogeneous Equation $(\lambda - D_m D_s^+)u = 0$ $(\lambda > 0)$ 155
5.14. Solutions of the Non-Homogeneous Equation $(\lambda - D_m D_s^+)g = f$ 159
5.15. Distributions of Various Quantities Associated with $\boldsymbol{x}^{(a)}(t)$ in a Regular Interval 162
5.16. Behavior of a Process at the Boundaries of a Regular Interval 164

Postscript 169