Contents

Preface *page* xiii

1 Introduction 1
1.1 Means and Ends 1
1.2 The First Regression: A Historical Prelude 2
1.3 Quantiles, Ranks, and Optimization 5
1.4 Preview of Quantile Regression 9
1.5 Three Examples 13
1.5.1 Salaries versus Experience 13
1.5.2 Student Course Evaluations and Class Size 17
1.5.3 Infant Birth Weight 20
1.6 Conclusion 25

2 Fundamentals of Quantile Regression 26
2.1 Quantile Treatment Effects 26
2.2 How Does Quantile Regression Work? 32
2.2.1 Regression Quantiles Interpolate p Observations 33
2.2.2 The Subgradient Condition 34
2.2.3 Equivariance 38
2.2.4 Censoring 40
2.3 Robustness 42
2.3.1 The Influence Function 42
2.3.2 The Breakdown Point 45
2.4 Interpreting Quantile Regression Models 47
2.4.1 Some Examples 48
2.5 Caution: Quantile Crossing 55
2.6 A Random Coefficient Interpretation 59
2.7 Inequality Measures and Their Decomposition 62
2.8 Expectiles and Other Variations 63
2.9 Interpreting Misspecified Quantile Regressions 65
2.10 Problems 66

3 Inference for Quantile Regression 68
3.1 The Finite-Sample Distribution of Regression Quantiles 68
3.2 A Heuristic Introduction to Quantile Regression Asymptotics 71
3.2.1 Confidence Intervals for the Sample Quantiles 72
3.2.2 Quantile Regression Asymptotics with IID Errors 73
3.2.3 Quantile Regression Asymptotics in Non-IID Settings 74
3.3 Wald Tests 75
3.3.1 Two-Sample Tests of Location Shift 75
3.3.2 General Linear Hypotheses 76
3.4 Estimation of Asymptotic Covariance Matrices 77
3.4.1 Scalar Sparsity Estimation 77
3.4.2 Covariance Matrix Estimation in Non-IID Settings 79
3.5 Rank-Based Inference 81
3.5.1 Rank Tests for Two-Sample Location Shift 81
3.5.2 Linear Rank Statistics 84
3.5.3 Asymptotics of Linear Rank Statistics 85
3.5.4 Rank Tests Based on Regression Rankscores 87
3.5.5 Confidence Intervals Based on Regression Rankscores 91
3.6 Quantile Likelihood Ratio Tests 92
3.7 Inference on the Quantile Regression Process 95
3.7.1 Wald Processes 97
3.7.2 Quantile Likelihood Ratio Processes 98
3.7.3 The Regression Rankscore Process Revisited 98
3.8 Tests of the Location-Scale Hypothesis 98
3.9 Resampling Methods and the Bootstrap 105
3.9.1 Bootstrap Refinements, Smoothing, and Subsampling 107
3.9.2 Resampling on the Subgradient Condition 108
3.10 Monte Carlo Comparison of Methods 110
3.10.1 Model 1: A Location-Shift Model 111
3.10.2 Model 2: A Location–Scale-Shift Model 112
3.11 Problems 113

4 Asymptotic Theory of Quantile Regression 116
4.1 Consistency 117
4.1.1 Univariate Sample Quantiles 117
4.1.2 Linear Quantile Regression 118
4.2 Rates of Convergence 120
4.3 Bahadur Representation 122
4.4 Nonlinear Quantile Regression 123
4.5 The Quantile Regression Rankscore Process 124
4.6 Quantile Regression Asymptotics under Dependent Conditions 126

4.6.1 Autoregression 126
4.6.2 ARMA Models 128
4.6.3 ARCH-like Models 129
4.7 Extremal Quantile Regression 130
4.8 The Method of Quantiles 131
4.9 Model Selection, Penalties, and Large-p Asymptotics 133
4.9.1 Model Selection 134
4.9.2 Penalty Methods 135
4.10 Asymptotics for Inference 138
4.10.1 Scalar Sparsity Estimation 139
4.10.2 Covariance Matrix Estimation 141
4.11 Resampling Schemes and the Bootstrap 141
4.12 Asymptotics for the Quantile Regression Process 142
4.12.1 The Durbin Problem 142
4.12.2 Khmaladization of the Parametric Empirical Process 144
4.12.3 The Parametric Quantile Process 145
4.12.4 The Parametric Quantile Regression Process 146
4.13 Problems 149

5 L-Statistics and Weighted Quantile Regression 151
5.1 L-Statistics for the Linear Model 151
5.1.1 Optimal L-Estimators of Location and Scale 153
5.1.2 L-Estimation for the Linear Model 155
5.2 Kernel Smoothing for Quantile Regression 158
5.2.1 Kernel Smoothing of the ρ_τ-Function 160
5.3 Weighted Quantile Regression 160
5.3.1 Weighted Linear Quantile Regression 160
5.3.2 Estimating Weights 161
5.4 Quantile Regression for Location–Scale Models 164
5.5 Weighted Sums of ρ_τ-Functions 168
5.6 Problems 170

6 Computational Aspects of Quantile Regression 173
6.1 Introduction to Linear Programming 173
6.1.1 Vertices 174
6.1.2 Directions of Descent 176
6.1.3 Conditions for Optimality 177
6.1.4 Complementary Slackness 178
6.1.5 Duality 180
6.2 Simplex Methods for Quantile Regression 181
6.3 Parametric Programming for Quantile Regression 185
6.3.1 Parametric Programming for Regression Rank Tests 188
6.4 Interior Point Methods for Canonical LPs 190
6.4.1 Newton to the Max: An Elementary Example 193
6.4.2 Interior Point Methods for Quantile Regression 199

6.4.3 Interior vs. Exterior: A Computational Comparison 202
6.4.4 Computational Complexity 204
6.5 Preprocessing for Quantile Regression 206
6.5.1 "Selecting" Univariate Quantiles 207
6.5.2 Implementation 207
6.5.3 Confidence Bands 208
6.5.4 Choosing m 209
6.6 Nonlinear Quantile Regression 211
6.7 Inequality Constraints 213
6.8 Weighted Sums of ρ_τ-Functions 214
6.9 Sparsity 216
6.10 Conclusion 220
6.11 Problems 220

7 Nonparametric Quantile Regression 222
7.1 Locally Polynomial Quantile Regression 222
7.1.1 Average Derivative Estimation 226
7.1.2 Additive Models 228
7.2 Penalty Methods for Univariate Smoothing 229
7.2.1 Univariate Roughness Penalties 229
7.2.2 Total Variation Roughness Penalties 230
7.3 Penalty Methods for Bivariate Smoothing 235
7.3.1 Bivariate Total Variation Roughness Penalties 235
7.3.2 Total Variation Penalties for Triograms 236
7.3.3 Penalized Triogram Estimation as a Linear Program 240
7.3.4 On Triangulation 241
7.3.5 On Sparsity 242
7.3.6 Automatic λ Selection 242
7.3.7 Boundary and Qualitative Constraints 243
7.3.8 A Model of Chicago Land Values 243
7.3.9 Taut Strings and Edge Detection 246
7.4 Additive Models and the Role of Sparsity 248

8 Twilight Zone of Quantile Regression 250
8.1 Quantile Regression for Survival Data 250
8.1.1 Quantile Functions or Hazard Functions? 252
8.1.2 Censoring 253
8.2 Discrete Response Models 255
8.2.1 Binary Response 255
8.2.2 Count Data 259
8.3 Quantile Autoregression 260
8.3.1 Quantile Autoregression and Comonotonicity 261
8.4 Copula Functions and Nonlinear Quantile Regression 265
8.4.1 Copula Functions 265

8.5 High-Breakdown Alternatives to Quantile Regression 268
8.6 Multivariate Quantiles 272
8.6.1 The Oja Median and Its Extensions 273
8.6.2 Half-Space Depth and Directional Quantile Regression 275
8.7 Penalty Methods for Longitudinal Data 276
8.7.1 Classical Random Effects as Penalized Least Squares 276
8.7.2 Quantile Regression with Penalized Fixed Effects 278
8.8 Causal Effects and Structural Models 281
8.8.1 Structural Equation Models 281
8.8.2 Chesher's Causal Chain Model 283
8.8.3 Interpretation of Structural Quantile Effects 284
8.8.4 Estimation and Inference 285
8.9 Choquet Utility, Risk, and Pessimistic Portfolios 287
8.9.1 Choquet Expected Utility 287
8.9.2 Choquet Risk Assessment 289
8.9.3 Pessimistic Portfolios 291

9 Conclusion 293

A Quantile Regression in R: A Vignette 295
A.1 Introduction 295
A.2 What Is a Vignette? 296
A.3 Getting Started 296
A.4 Object Orientation 298
A.5 Formal Inference 299
A.6 More on Testing 305
A.7 Inference on the Quantile Regression Process 307
A.8 Nonlinear Quantile Regression 308
A.9 Nonparametric Quantile Regression 310
A.10 Conclusion 316

B Asymptotic Critical Values 317

References 319
Name Index 337
Subject Index 342